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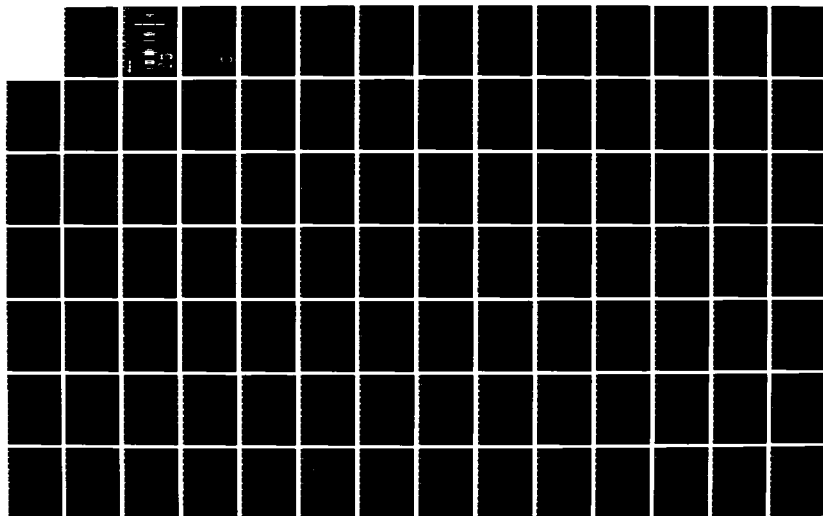
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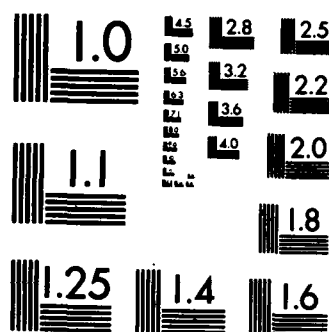
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FINAL REPORT  
FOR  
ADAPTIVE CANCELLATION TECHNIQUES

Submitted to

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Electromagnetic Systems Division  
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# ABSTRACT

An adaptive signal canceller has been evaluated for the enhancement of pulse signal reception during the transmission of a high power ECM jamming signal. The canceller design is based on the use of DRFM (Digital RF Memory) technology as part of an adaptive multiple tapped delay line. The study includes analysis of relationship of tap spacing and waveform bandwidth, survey of related documents in areas of sidelobe cancellers, transversal equalizers, and adaptive filters, and derivation of control equations and corresponding control processes. The simulation of overall processes included geometric analyses of the multibeam transmitting antenna, multiple reflection sources and the receiving antenna; waveforms, tap spacings and bandwidths; and alternate control algorithms. Conclusions are provided regarding practical system control algorithms, design characteristics and limitations.

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## SECTION I

### INTRODUCTION

In this study, an adaptive signal canceller is evaluated for the enhancement of pulse signal reception during the transmission of a high power ECM jamming signal. The canceller design is based on the use of DRFM (Digital RF Memory) technology with application to ECM systems in general and the AN/SLO-32 in particular. The study complements work funded by Navelex (Contract N000039-81-C-0361) which included the breadboarding of a coherent three output port DRFM and the successful demonstration of digital delays to cancel a broadband signal.

ECM lookthrough refers to the process of detecting the presence of an enemy radar signal simultaneous with full-power jamming with own ECM transmitter. The ECM system receives this signal through a pre-selected antenna port. With the AN/SLO-32, semi-omni antennas are used. ESM lookthrough is considered to be a different problem, that of original detection and follow-up parameter measurement for all emitters in the environment. For the SLO-32 and other modern ECM systems with "wide-open" angle and frequency capability, the ESM lookthrough problem has other unique requirements not addressed in this study.

An overview of the study is presented in Figure 1-1. Inputs to the study include the various concepts and practical results of sidelobe canceller and adaptive filter technology plus the coherent DRFM experience and technology. The canceller approach is based on a philosophy described in reference 2, briefly diagrammed on figure 1-1, and outlined as follows:

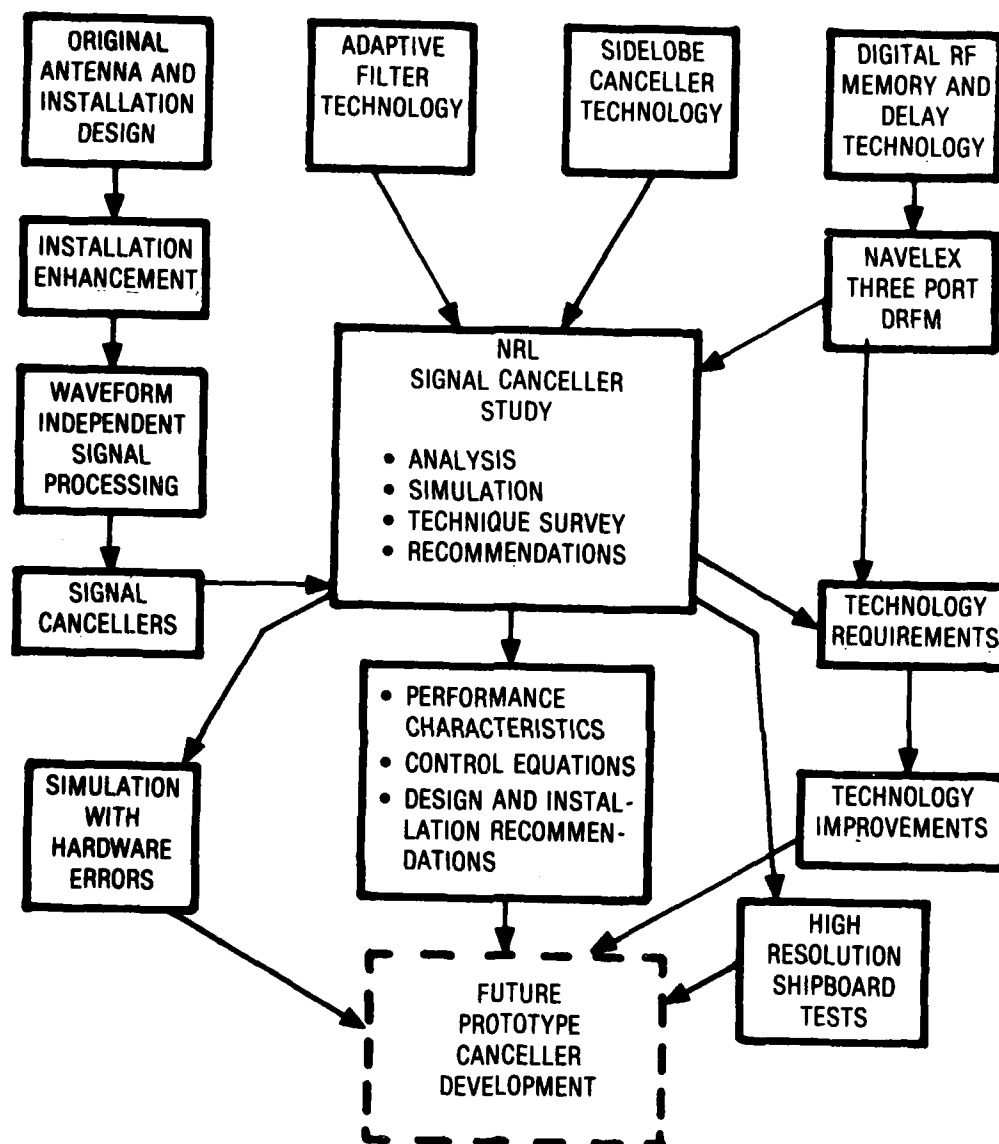


FIGURE 1-1 RELATIONSHIP OF STUDY TO ECM LOOKTHROUGH PROBLEM

1. Design good isolation into original antennas and installation;
2. Enhance installation with absorber, baffles, chokes and other passive improvements;
3. Use signal processing techniques which are independent of ECM waveform, e.g. cancellation of own transmitter signal.

This study specifically addresses time domain cancellers based on the premise that fewer channels would be required than in a frequency domain implementation.\* However, the performance limitation and control process algorithm results from this study are considered useful for either time domain or frequency domain implementation of a canceller. The premise of fewer time domain channels and the relative number of channels must be verified before final design and cost trade-offs are made for a production prototype canceller.

\*Regarding the use of signal cancellers, it is recognized that either frequency domain or time domain channelization will accomplish the necessary cancellation, and that (for complete time and frequency domain coverage) the same number of channels will be required in either case. It is argued in reference 2 that the ECM lookthrough problem is probably characterized by significant reflected signals from only part of the time domain defined by the region from 1) the time for the shortest coupling path between transmit and receive antennas, to 2) the time for the longest two way signal propagation to the physical extremes of the ECM platform. Based on this premise, ECM cancellers may require fewer (but wider bandwidth) channels in the time domain than in the frequency domain.

The primary purpose of this study was to provide analysis and simulation leading directly to signal canceller performance prediction and control algorithms for future implementation with ECM systems in general and the AN/SLQ-32 in particular. Study efforts included analysis, simulation, technique survey, and preparation of recommendations. The study was limited to signal canceller approaches based on multiple delays in the canceller paths to achieve broadband effects against a wideband jamming signal, and further to the use of a multiple port DRFM as the delay medium.

Conclusions include approximate parametric performance equations, recommended control equations, recommendations on shipboard measurements, design, installation, further simulation with modeled hardware errors, and in combination with the corresponding Navelex work, some technology suggestions.

The study includes a careful review of similar techniques in radar and communications applications. These include sidelobe canceller, adaptive antennas and filters, and transversal equalizers.

## SECTION II

### TECHNICAL DISCUSSION

#### 2.1 ANALYSIS OVERVIEW

The purpose of the analysis was to derive algorithm equations and to explore performance of adaptive delay line cancellers as function of bandwidth and tap spacing. All analysis was based on variants of the fundamental canceller as shown in figure 2-1. The transmitted signal  $u(t)$  is radiated from the transmit antenna and coupled into a tapped delay line. The transmitted signal is reflected from a number of reflection points at various time delays and complex amplitudes (amplitude and phase). The receive antenna captures both the desired signal  $s(t)$  and the reflected transmit signals. The tapped delay line provides delayed copies of  $u(t)$  which are weighted with controllable complex coefficients and summed in the receiver along with the antenna inputs and the receiver noise  $n(t)$ . When a DRFM or DRFM delay technology is used as the delay medium, the output to the transmitter is also taken from the first tap of the DRFM in order to be coherent with the other outputs.

Appendix A presents the derivation of control equations based on a Least Mean Squares matrix inversion approach. This approach essentially parallels the classic early sidelobe canceller work (Refs 3 and 4). These equations are implemented in a simulation and the performance is tested as described later.

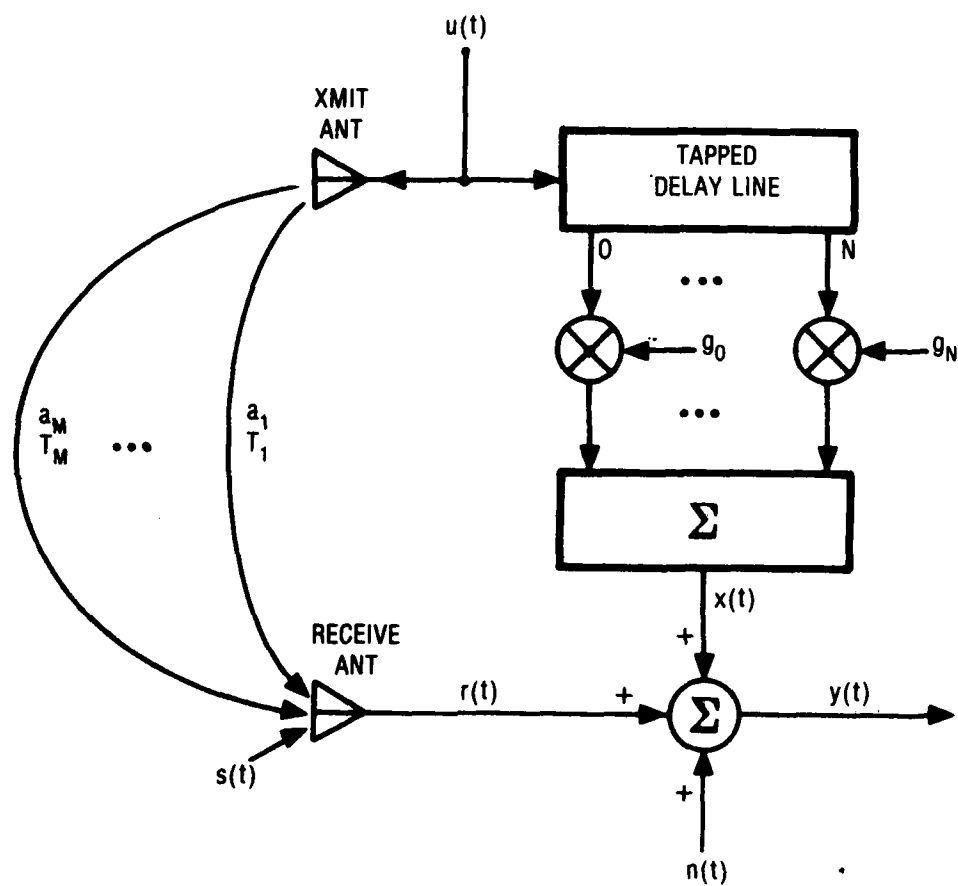


FIGURE 2-1 INTERFERENCE CANCELLATION CIRCUIT

Appendix B presents the derivation of control equations based on an iterative Least Mean Squares gradient approach, essentially the same as that used in adaptive filters (Refs 9,10,11). These equations are also implemented in a simulation and the tested results are reported in a later section.

Appendix C presents an intuitive understanding of the ECM lookthrough reflection environment and the way in which a tapped delay line canceller is matched to that environment.

Appendix D presents an analytic set of models for performance of cancellers with a particular class of signals. The results of these calculations are presented in a later section.

Appendix E describes the simulations used during this study.

Appendix F describes the bandpass filter model used with the simulations.

Appendix G contains example simulation plots of video before and after cancellation.

## 2.2 SIMULATION OVERVIEW

A major part of this study effort was devoted to building and using a computer software simulation of a complex reflection environment and the various canceller control processes. This simulation work emphasized use of pulsed waveforms (e.g. a narrow spike gated from a wide bandwidth DRFM), radiated from a multi-beam transmitter antenna (9 to 30 elements) and returned from a distribution of point reflections (3 to 20 scatters, 7 typical) to a bandwidth limited single antenna port, single channel receiver (Butterworth two pole filter). Cancellation is modeled for various combinations of control algorithms and tap spacings. Time, amplitude and phase are modeled for each tap. The simulation presents time domain view of composite video before and after cancellation. It was written in BASIC to run on HP 9830 and Z-80 based desktop microcomputers and did not contain hardware error or non linear models.

The philosophy adopted in constructing the simulation was to start with the simplest embodiment of the essential elements. The "system" to be modelled consists, in part, of the jammer transmitter and antenna, a set of scatterers illuminated by the radiated "reference" signal, and the receiver which picks up the undesired "raw signal" return resulting from the combination of waves via all the paths. Although undesired, this raw signal must be the source of the information needed to cancel it, requiring this miniature radar to sense and, in some degree, resolve the set of scatterers. However, the resolution requirement seems intuitively not to be absolute but rather related to the effective jamming signal bandwidth as established by the generation process and any filters present in the transmitter and receiver. Thus the complexity of a

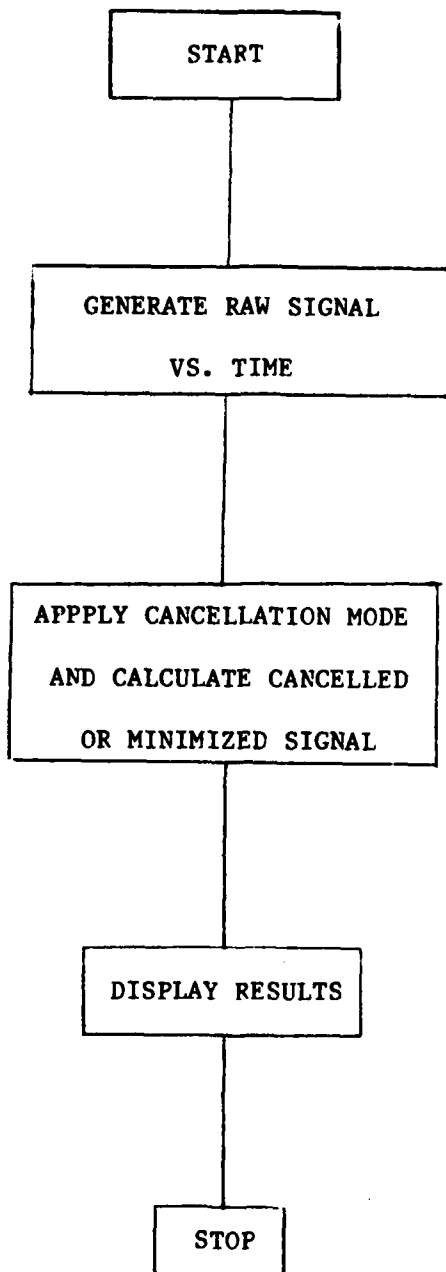
large set of scatterers concentrated in range would tend to be lost in a narrow band system. This loss would be counterproductive in one sense: less information about the scatterers would be available from the received signal. On the other hand, there should be a compensating simplification of the information needed to control the cancellation.

As to complexity, it seems intuitively clear that, ignoring questions of S/N, multiple samples, measurement errors, and the like, a single "narrowest" pulse signal should provide a good handle on the problem, while meeting the criterion of simplicity. In essence, a more complex signal of the same bandwidth but greater energy would provide reduced random measurement errors caused by noise, but this effect could be readily predicted and need not be modelled initially. (Measurement errors have not yet been incorporated.)

The model design is a straightforward representation in that the raw signal is obtained by linear summing of the component waves which travel via all the different paths from each transmitting antenna array element via each scatterer to the receiving antenna. The receiving antenna could also be an array, at the cost of multiplied computer time and storage to handle the larger multiplied number of paths; for reasonable times on a small machine a single receiving element is used. This approach includes the essential elements of the problem including the effect of the array pattern to whatever extent it resolves the scatterers in angle, and near-field effects as well, in case the scatterers are at close range.

As to bandwidth limitation occurring in the system filters, the model takes advantage of linearity to place all the filtering at one point - just ahead of the transmitting antenna. Receiver filtering as such would operate with a very complicated input signal, making the output difficult to compute. This problem is avoided by computing and storing the "reference waveform" generated in a single filter driven by a single pulse. A previously developed program, FILT, is used to compute this waveform. The bandpass filter is defined by its bandwidth, number of poles, and passband ripple. It is driven by an AM (only) pulse with piecewise linear envelope, which may have a carrier offset from the filter center frequency. (However, no offset was used in this application.) This program applies the La Place transform transient analysis to the low-pass prototype filter.

The general structure of the model is shown below. The raw signal generation process is the same for all versions of the model. Beyond this point, the various concepts of cancellation control give rise to a series of distinct variations which are not conveniently combined in one overall model, due in part to the conflict between program size and array storage requirements.



The elementary "single point" cancellation concept uses a replica of the radiated signal from a delay line superimposed on the raw signal return. At any one particular non-zero "key" point (in time) of the delay line signal, a complex "weight" exists by which this signal can be multiplied to obtain an exact cancellation of the raw signal and the delay line output at this time, the required weight is simply their complex ratio. Using this same weight, the resulting output can be found as a function of time\*.

This single-point cancellation is the basis for the "sweep" concept of cancellation control (model version MST1, "MST" for "mast") in which outputs from a series of delay line taps at a given spacing in time are successively added with appropriate weights to cancel a corresponding series of points at the same spacing. The subtotal at each point includes the raw signal and contributions from all those components previously added. At each tap, the "key" point of the delayed wave emerges at the chosen "cancel time". This favors a key point which is a leading edge "shoulder" - i.e., a high point preceded only by low points. Such a point would tend to be one preceding the peak, while the peak itself would further clutter the later parts of the undesired signal and tend to cause a "bushy tail" effect toward the end of the sequence.

\*Two considerations of concern can be quickly disposed of. (1) The delay line output is, strictly, the inverse Fourier transform of the product of the signal Fourier transform and the delay line transfer function. However, if the delay line is non-dispersive, it is necessary only to replace it by a time-delay. (2) There is no need to account for explicit phase shift in the delay line (many thousand radians); this would merely result in changing the angle of the required complex weight.

A variation on this concept is to under-correct somewhat to reduce the effect of the current addition on the preceding cancellations. A tailoring of this under-correction might help to counter end of signal buildup. A second variation is to recycle the series of cancellations. A given set of taps can be divided into interlaced sets which are "scanned" alternately. All of these except tailoring of under-correction have been used in the "sweep" model.

The simultaneous equation concept (version MST3) provides one measure of what is theoretically possible. Given a set of  $n$  taps and matching raw signal "cancel time" points, a set of simultaneous equations results from requiring the sum of the raw signal and all the tap outputs to be zero at each of the given time points. With known values of the raw signal at these points and the known tap outputs at each tap at each time, these equations can be solved by matrix inversion to give the  $n$  weights ("weight vector"), and the "cancelled" signal can be computed at all time points. This should show deep nulls at all cancel times. Intervening points are given no weight in this solution and might remain fairly high except that bandwidth limiting implies a limit on spikiness relative to a given tap spacing. With the raw signal vector  $Y$  and delay line output matrix  $u$ , a formal statement of this solution requires the weight vector  $g$  to satisfy the matrix equation

$$Y + ug = 0, \text{ or}$$

$$g = -u^{-1}Y$$

In a straightforward manner, the model constructs the matrix  $u$ , inverts it, and forms the (negative) product to get the weight vector and then sums the weighted tap outputs with the raw signal.

Another control concept (version MST4) is the matrix inversion approximation to the Least Mean Square (LMS) criterion; i.e., minimum mean power over a set of  $m$  time points using weighted signals from  $n$  taps, where  $m$  is greater than  $n$ . If  $m = n$ , LMS is identical to the simultaneous equation concept. The  $n$  taps emit delayed signal outputs at the  $m$  cancel times and these outputs constitute an  $m \times n$  matrix  $u$ . The raw signal outputs at the  $m$  times form a vector  $Y$ . With the optimum weight vector  $g$ , the LMS output vector (i.e., the outputs at the  $m$  time points) resulting from summing the raw signal and all the weighted tap outputs is

$$Y + ug$$

The LMS criterion requires that  $g$  satisfy the equation

$$(u^*T_u)^{-1} u^*TY + g = 0$$

The model MST4 methodically evaluates the (negative of the) first term to get the weight vector  $g$ . It is not necessary to store the conjugate transpose  $u^*T$ ; interchanging indices and sign reversal of the imaginary part are performed in "pick-up" for the two products  $u^*T_u$  and  $u^*TY$ .

Both MST3 and MST4 require inversion of a matrix, either the delay line output matrix itself,  $u$ , or the product  $u^*T_u$ , derived in either case from the stored reference waveform. In general, this waveform is complex. However, the waveforms actually used are real, and the assumption that delays at the taps are integral multiples of the carrier frequency causes no loss of essential generality. Under these conditions, the tap outputs are real and therefore  $u$  is real. Therefore the inversions, and the product  $u^*T_u$  obtained prior to inversion in MST4, can be found by calculations with reals only. This reduces both running time and storage requirements. Work with MST3 had been completed before this was realized, but modification of MST4 was made to reduce these calculations to reals only, and designated MST4RL. Its identical results were demonstrated.

Perhaps the most interesting concept for cancellation control (version MSTGR) is the well-known gradient algorithm or method of steepest descent. The iterative process starts with an arbitrary weight vector (e.g., zero) and at each iteration updates the weight vector by adding a corrective vector in a manner analogous to negative feedback, including a negative "gain factor". Each element of this corrective vector applies to a particular tap, and, excluding the negative factor, is an integral (or finite summation at intervals  $\Delta t$ ) over the whole signal duration. The integrand is the product of the residual, incompletely cancelled signal from the last cycle and the unweighted tap output. Unlike the other processes, this one has no exit, but runs indefinitely unless interrupted. It exhibits a time constant or rate, and may be unstable, depending on the gain factor.

One final model version MSTX does not represent a cancellation control concept but calculates what happens when the transmitted waveform is changed after the weight vector has been adjusted for an initial "calibrating" waveform. In this model, fixed weights are read from a data table generated in running the "calibrating" model, e.g., the matrix/LMS model, MST4 or MST4RL, with the identical configuration of scatterers and antennas and with the same set of taps. Thus MSTX poses its own problem (the raw signal) and tests the solution supplied by another model.

## 2.3 ANALYSIS AND SIMULATION RESULTS

In Appendix D, four mathematical models were derived for the following signal cancellation cases with slow frequency modulation on a cw signal and a single environment reflection point:

- o Model  $C_1$  Single tap canceller - band edge performance
- o Model  $C_2$  Single tap canceller - average performance over band
- o Model  $C_3$  Two tap canceller - band edge performance
- o Model  $C_4$  Two tap canceller - average performance over band

These models are plotted in figure 2-2 for 500 MHz bandwidth using the upper abscissa and 250 MHz using the lower abscissa. Curves  $C_1$  and  $C_2$  indicate how close a time match would be required to achieve the cancellation bandwidth with one tap. Curves  $C_3$  and  $C_4$  indicate how closely spaced two taps would have to be if centered on the single reflection point. All of these performance models suggest that to a first order approximation, a given level of cancellation performance can be achieved by maintaining a tap spacing bandwidth product less than a specific value. Using  $C_4$  for 500 MHz bandwidth taps should be as close as 0.8 ns to achieve 20 dB cancellation, and for 250 MHz bandwidth taps should be as close as 1.6 ns. In general as reported later, the simulations bear this out for pulsed signals at 500 MHz, 250 MHz and other bandwidths down to and including 30 MHz. Therefore equation D -14 (Appendix D) for  $C_4$  is suggested as an empirical model for determining canceller tap spacing to achieve a given bandwidth of performance.

The simulation was first configured (see Appendix E for configuration MST1) to experiment with the sweep method of control as previously described. This would lead to a relatively simple control hardware methodology as follows. Control hardware would use a pulse leading edge detector in combination with an amplitude threshold, amplitude encoding and fine time encoding. Taps would be adjusted consecutively from smallest delay to largest delay. With each successful adjustment, the leading edge detection of the composite RF waveform would move toward the longest delay. When the entire reflection region was "swept", the adjustment would be complete. The results of sweep control are presented in figure 2-3 representing 21 cases at 500 MHz bandwidth and 5 cases at 250 MHz bandwidth. Performance better than 20 dB was marginal.

The simulation was next configured for a simultaneous equations solution (see Appendix E for configuration MST3). This solution used known pulse waveform characteristics and detailed knowledge of amplitude and phase of the composite reflected transmitter signal before cancellation in order to force the after cancellation signal to zero at equally spaced points (tap spacing). Although probably impractical to implement in control hardware, this model of cancellation was developed as a baseline to compare with that achieved using practical control algorithms. Figure 2-4 presents the simulation results indicating the general bandwidth tap spacing constraints of cancellation performance.

The simulation was then configured to implement the Least Mean Square (LMS) matrix inversion equation model as described in Appendix A. Results are presented in figure 2-5. Excellent theoretical performance is shown. As previously stated, this model is essentially the same as that used for sidelobe canceller designs (Refs 3 and 4).

The simulation was configured to implement the LMS gradient equations from Appendix B (Simulation configuration MSTGR). This implementation is comparable with the adaptive filter approach (Refs 9,10,11). Six cases were run at 2.4 ns tap spacing for a 250 MHz 4.5ns pulse with varying point reflector geometries. All cases converged to the approximate performance of the corresponding LMS matrix inversion case. Three additional cases were run for a 500 MHz, 3.55 ns pulse and converged to comparable LMS matrix performance. Figure 2-6 depicts the rate of convergence for several cases as a function of the gain factor applied to the gradient process. With sufficient gain the process converges in less than 10 iterations.

It is clear that for matrix inversion, large numbers of taps require inversion of large matrices with the corresponding problems of coefficient dynamic range, data errors precluding solution, and expensive high speed data processing. The widely used solution to this dilemma is the LMS gradient approach originated by Widrow ( Refs 9,10,11).

To test the possibility of calibrating the canceller with one waveform and subsequently radiate another with the same or less bandwidth, the simulation version MSTX (See Appendix E) was used. A calibration was established for a 250 MHz 2.4 ns tap case, and 7 variations in pulse waveforms were passed through the simulated cancellation network without control change. The cancellation continued to be effective with only a few dB of degradation. More cases should be run for this situation.

To test the cancellation response to a noise like waveform, one case was designed with two narrow pulses in cascade with a 90 degree phase difference between them. The transition region during which one decayed and the other increased in amplitude carried the typical phase shift which would be expected to exist in the bandpass canceller channel. The cancellation also worked this with this signal although tighter tap spacing would be required to achieve the same level of control as with straight pulse signals. Again, more simulation is desired for noise waveforms. Specifically, a noise waveform from a quadri-phase RF modulator is relatively straightforward to generate and include in a future simulation.

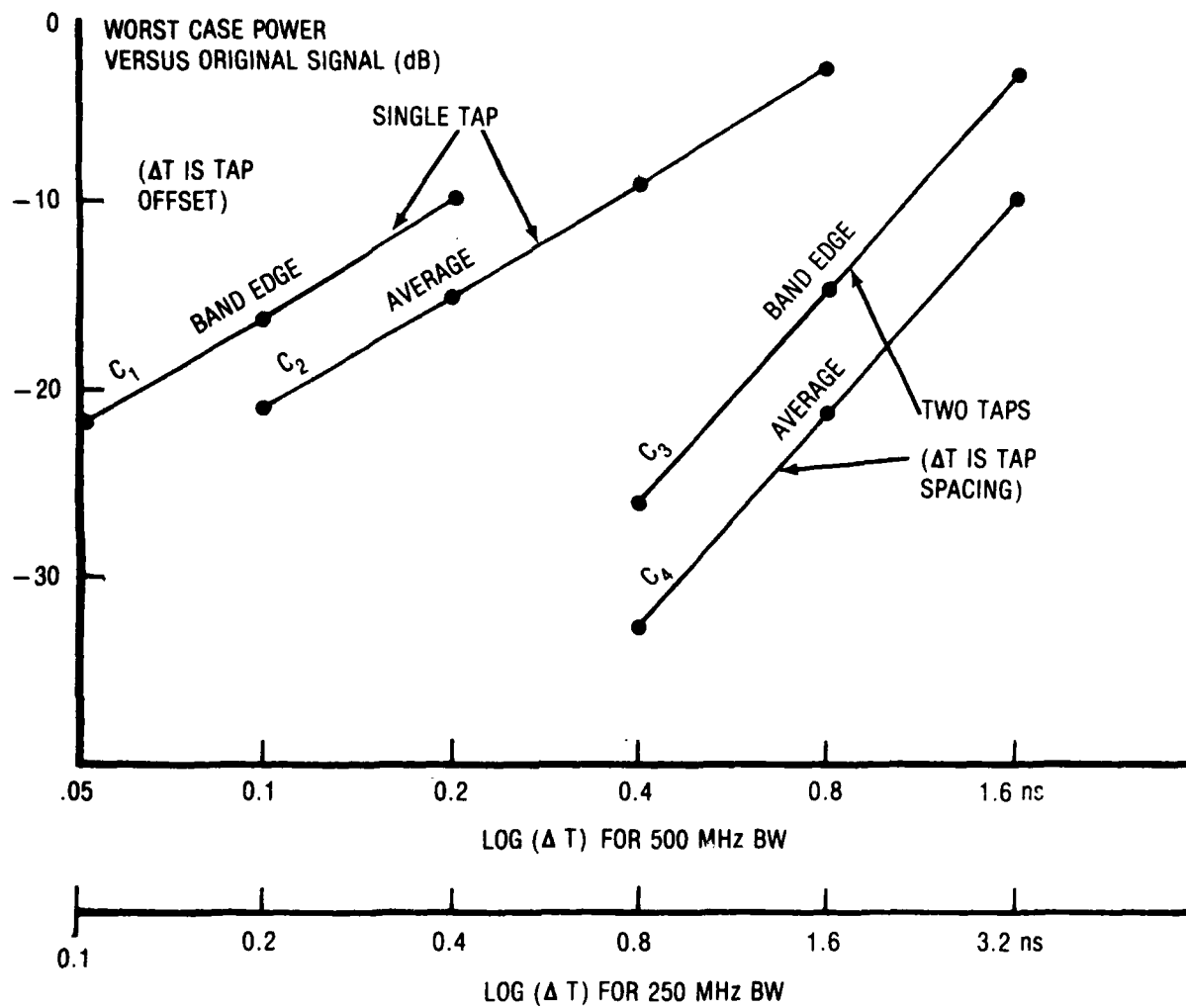


FIGURE 2-2 SIMPLE CONTINUOUS SIGNAL MODEL  
CANCELLATION PERFORMANCE

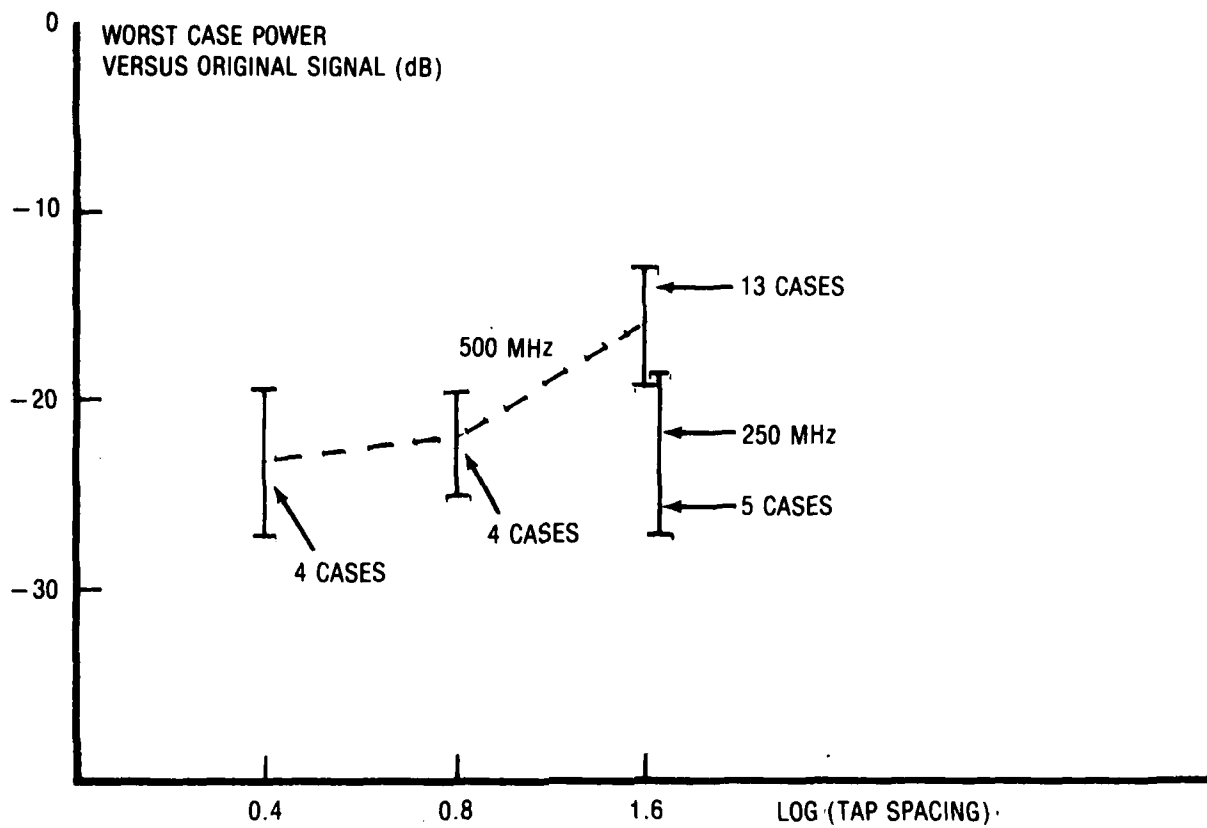


FIGURE 2-3

FIGURE 2-3 SHORT PULSE SIGNAL SIMULATION PERFORMANCE  
SWEEP CANCELLATION TECHNIQUE

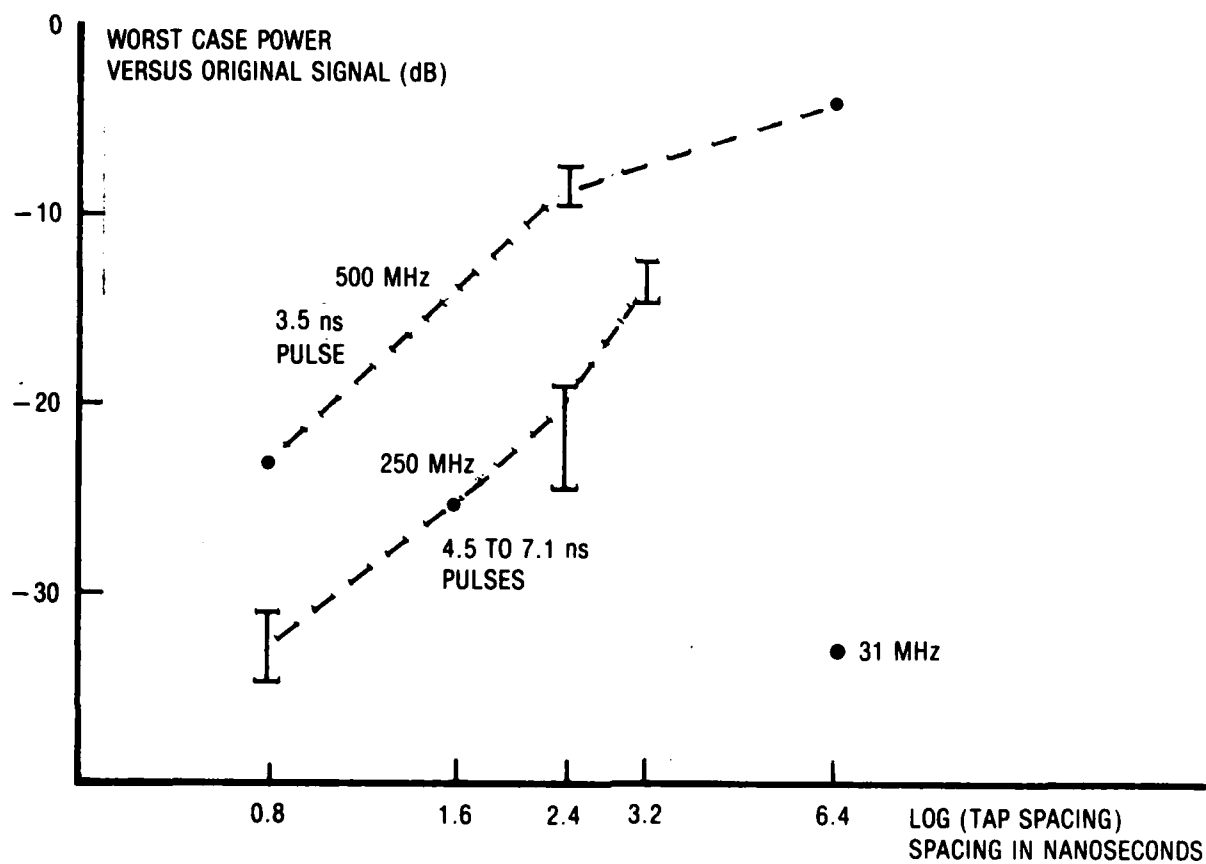


FIGURE 2-4 SHORT PULSE SIGNAL SIMULATION- SIMULTANEOUS  
EQUATION CANCELLATION TECHNIQUE

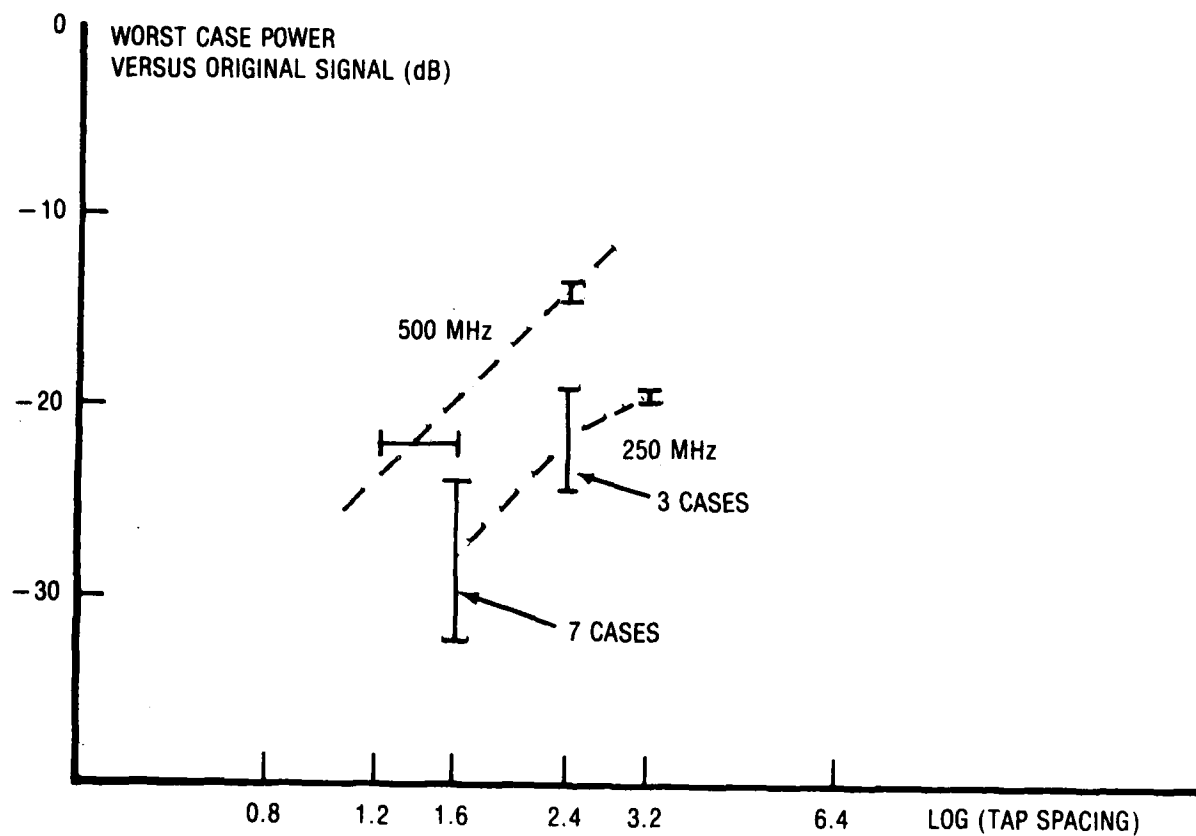


FIGURE 2-5.

FIGURE 2-5 SHORT PULSE SIGNAL SIMULATION-MATRIX  
LEAST MEAN SQUARE TECHNIQUE

WORST CASE POWER  
VERSUS ORIGINAL SIGNAL (dB)

FIGURE 2-6

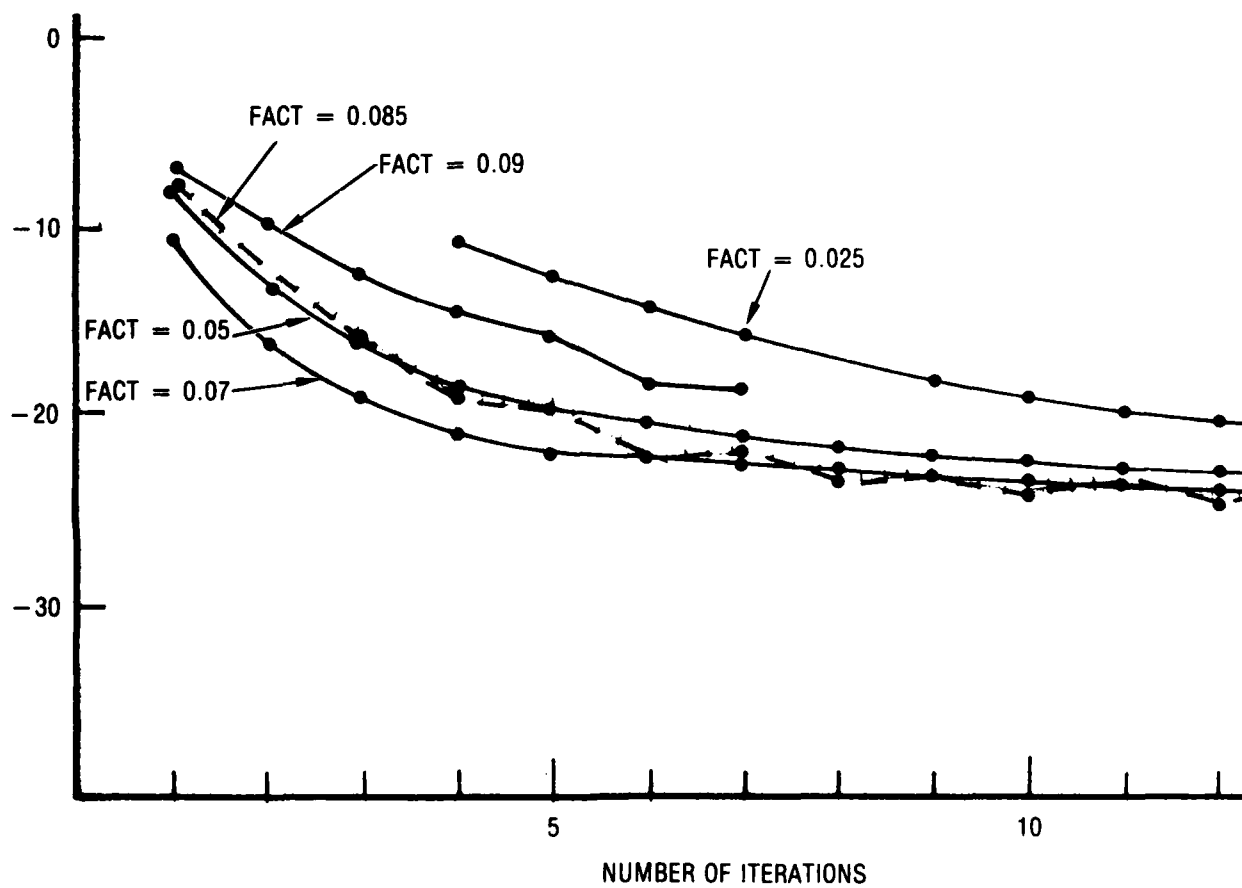


FIGURE 2-6 GRADIENT LMS CONTROL - GAIN FACTOR STUDY

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### SECTION III

#### RECOMMENDATIONS/CONCLUSIONS

For a linear system, the product of bandwidth and tap spacing governs the level of cancellation performance. For 20 dB of worst case interference residue after cancellation, the bandwidth tap spacing product should be less than 0.4. When compared with antenna technology, this is a slightly tighter constraint than that required to keep grating lobes out of real space. The time bandwidth constraint is also compatible with the Nyquist criterion for sampling a waveform.

For a process limited to 30 MHz of bandwidth, tap spacings of 12 nanoseconds will provide a nominal 20 dB of cancellation. For an expected shipboard environment with transmitter signal leakage and reflection paths of lengths less than 20 feet combined with another 20 foot group of paths at much larger distances, this would correspond to a set of 22 taps. For wider bandwidths, proportionally more paths are required.

For a bandwidth of 500 MHz, this environment would require a tap spacing of 0.8 ns for at least 20 dB of cancellation. For this wideband situation, the use of custom hybrid DRFM circuits for each channel tap would reduce the per channel costs significantly.

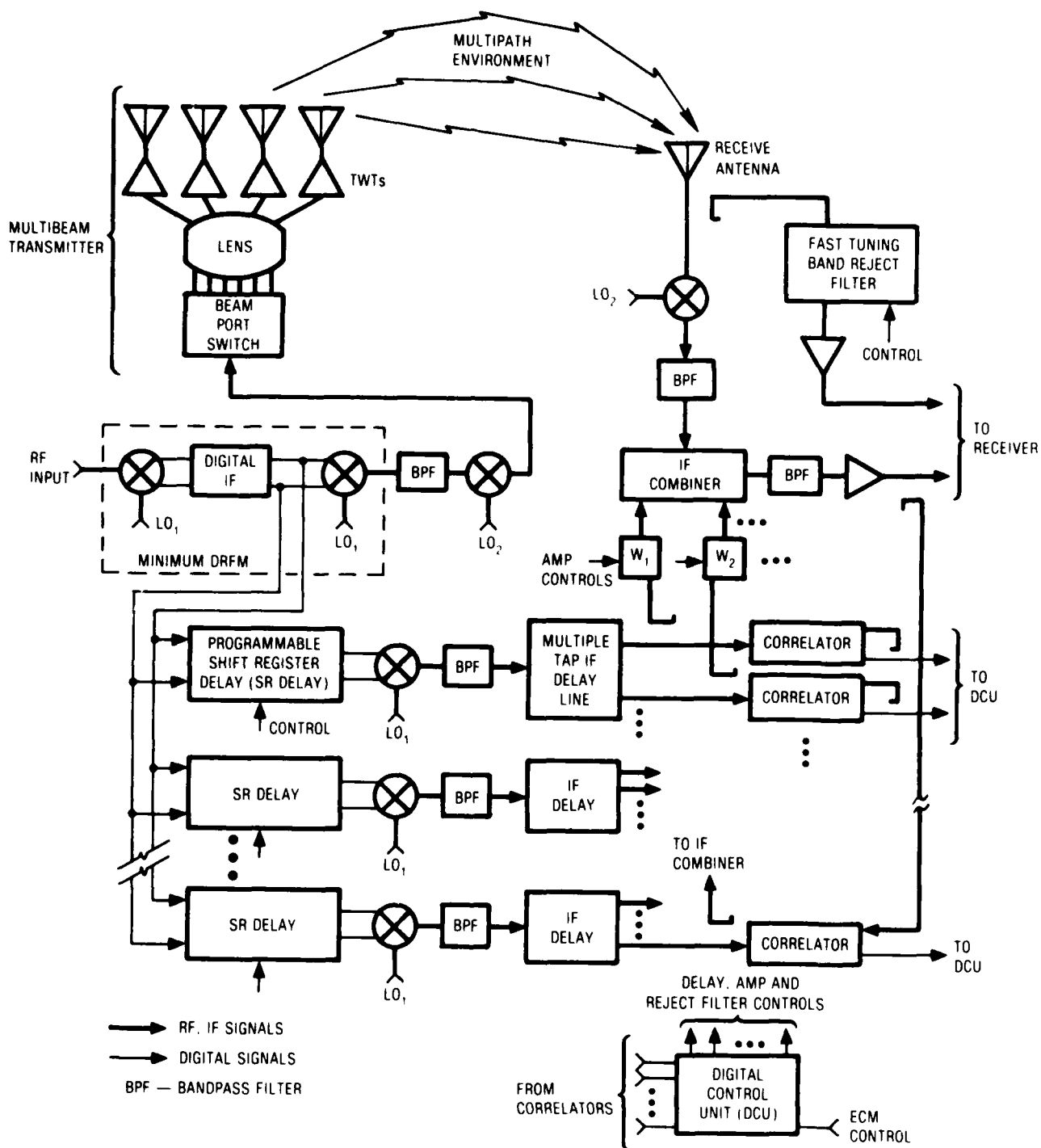


FIGURE 3-1 EXAMPLE ECM LOOKTHROUGH SUBSYSTEM

From the simulation results, levels of cancellation performance are comparable for direct matrix inversion and LMS iterative algorithms. General experience as described in the reference literature (Ref 4), indicates that the direct matrix approach is susceptible to poorly conditioned matrices (e.g zero elements or high dynamic range of elements) with the practical constraint that the more the number of taps in a system, 1) the larger the matrix, 2) the more precision required of inversion calculation, 3) the higher the probability of poor conditioning and 4) the less likely that a system will always converge to a useful cancellation performance level. The LMS iterative approach is therefore most strongly recommended.

Adaptive cancellers should be capable of recalibrating for ECM transmission against a particular emitter in a time approximately commensurate with one pulse to pulse time interval for that emitter. This would assure continuous lookthrough if the emitter should move one transmitter beamport in azimuth from that with which it was previously engaged. Simulation results with the LMS iterative algorithm indicate that canceller can converge to a useful level of performance in a sequence of about 5-10 steps.

The study includes the consideration of implementation of the control process as an ECM subsystem. Application and installation are considered primarily for shipboard use but airborne applications are not ignored. For the primary customer interest (AN/SLO-32), the canceller subsystem would consist of two identical outboard units, each installed in the corresponding repeater chain. Figure 3-1 shows an example ECM look through subsystem using the canceller concept and DRFM (Digital RF Memory) technology. Transmission would be from a multi-beam transmitter. Reception would be from a wide angle, wide band single port antenna. For wide open band reception, two receiver channels would be used. The first would be band limited and have the canceller features over a defined bandwidth. The second would be band notched with an agile band rejection filter, for example a varactor tuned filter. Canceller and band rejection filter bandwidths would be comparable to the largest ECM transmission bandwidth. Transmitted signals would be emitted from a DRFM and upconverted to the transmission frequency. In the shipboard application, either two DRFM's would be used, one associated with each shipside and canceller or a special minimum DRFM like structure would be used in each canceller. The minimum unit would encode the RF into digital IF signals which are reformed into RF with minimum delay and transmitted. The digital IF signals are routed to programmable shift register delays and reconstituted as IF signals and passed through multiple tap delay lines. As was demonstrated in the complementary contract efforts funded by Navelex, the digital signals can be reconstituted as coherent copies of the transmitted signals and used for cancellation. (Multiple levels of encoding must be used with the digital IF in order to hold the spurs to required levels for practical cancellation.) Each IF tap is weighted (in general with amplitude and phase control) and coherently added to the received signal path. The sum of all contributions is routed to a receiver

and coupled back to correlators (correlation mixers and encoding). Correlation data between taps and the receiver composite signal are used by the digital control unit to adjust the overall cancellation process. Shift register delays are used to adapt to the delays of major ECM reflection sources (e.g. a mast) in a given ECM installation. Multiple IF tap time intervals are set very close together to cover the total time delay range for all reflections from that source. Tap spacing results from this study are indicative of the IF tap spacing requirements for a given canceller bandwidth. If IF taps are spaced even closer together than previously described in this report, each tap could be weighted in amplitude only (not phase). This option may be worth detailed consideration.

Each emitter pulse would be engaged by direction from the normal ECM control process, typically tracker or transponder controlled to emit a jamming waveform in a specific direction aimed toward a specific emitter or a group of emitters. A digital control unit (DCU) would adjust the canceller for each transmission situation, maintaining past adjustments and periodically updating the calibration using a burst of ECM transmission.

The canceller would self-adjust all of its delayed signal paths to cancel extraneous reflected jamming signal transmissions over the entire repeater bandwidth. This effectively opens the repeater receiver channel to any arriving signal in the system band. The canceller process adapts to each jamming waveform and radiating beam port over a period of microseconds to milliseconds depending on implementation of control process, and it changes control parameters in synchronization with each change of jamming waveform, thereby engaging multiple emitters as required. Canceller process can be

calibrated with a different waveform than is used for jamming transmission, making it possible to calibrate without observation (out of band for the radar victim or mismatched to victim radar signal processor).

The following specific near term recommendations are emphasized:

- 1) The existing simulation should be expanded to include modeling of realistic hardware element errors.
- 2) A shipboard dockside environment test should be run including the demonstration of multiple tap time cancellation performance over a short but conclusive time interval. The time interval should be moved during the test to demonstrate control over the entire shipboard reflection domain and to assure that the time domain of reflections is not beyond expectations. A minimum test configuration would have 3 to 4 taps.
- 3) The VHSIC technology should be expanded to include the shift register/mixer combinations necessary for large numbers of DRFM output taps. Also consider the use of digital IF and correlation logic. GaAs should be considered including GaAs CCD implementations.
- 4) For wide open reception, the existing fast tuning filter technology (varactors) should be expanded.

APPENDIX A

DERIVATION OF CONTROL EQUATIONS

BASED ON A LEAST MEAN SQUARES

MATRIX INVERSION APPROACH

## APPENDIX A

### INTRODUCTION

An adaptive cancellation scheme is being investigated to determine the feasibility of suppressing the leakage signal coupled from a nearby jammer's transmit antenna into the protected receive antenna.

The general configuration of the cancellation scheme is shown in figure A-1. The jamming signal  $u(t)$  is coupled into the receive antenna after having been reflected from various point reflectors. Let the number of such reflectors be  $M$ . Each reflected signal  $u_{ri}(t)$  will have some unique complex reflection coefficient  $a_i$  and time delay  $T_i$ . That is  $u_{ri}(t) = a_i u(t - T_i)$ .

It is assumed that a sample of the jamming signal is available to the adaptive nulling network. This sample is input into a tapped delay line with  $N$  taps of equal spacing  $T$ . The output of the taps is weighted and summed to form the cancellation signal  $x(t)$  which is added to the receiver output signal.

The receiver signal  $r(t)$  will consist of the reflected signals, some desired signal  $s(t)$  and additive noise  $n(t)$ . The canceller output will consist of the received signal plus the cancellation signal  $x(t)$ .

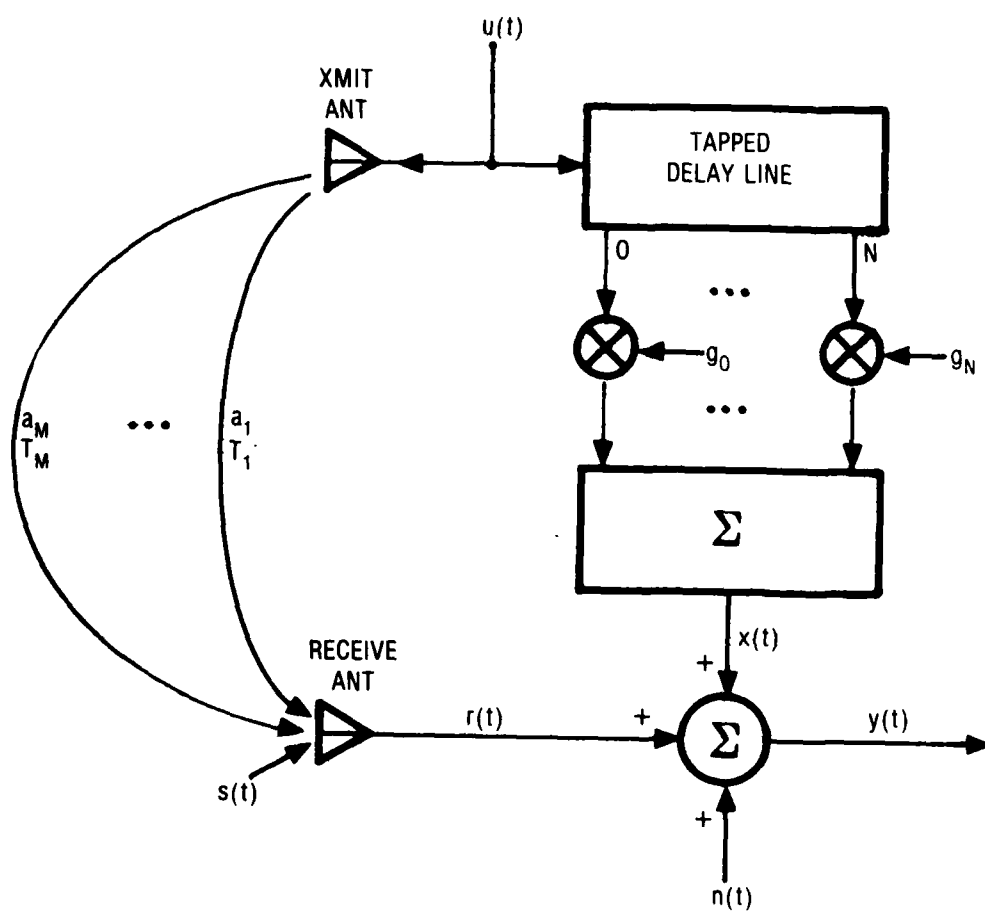


FIGURE A-1.

FIGURE A-1 INTERFERENCE CANCELLATION CIRCUIT

The basic idea of the canceller is to determine the weights  $[g_1, g_2 \dots, g_N]$  which best minimize the amount of reflected jamming signal at the canceller output. For example, if the time delays of the reflected signals corresponded exactly to the tap spacing of the delay line, then making  $g_1$  equal to  $a_1$  would produce perfect cancellation. On the other hand, if the time delays are randomly spaced between the tap values, perfect cancellation is generally not achievable unless the jamming signal has zero bandwidth.

The degree of cancellation that is achievable will depend on the characteristics of the jamming signal, the number and spacing of the taps in the delay line and the characteristics and distribution of the point reflectors.

#### APPROACHES

There are two approaches to analyzing the tap spacing problem, one in the frequency domain and one in the time domain. There is also a matter of treating the interference signal as a deterministic or a stochastic problem. For the present, the interference will be treated as a deterministic problem whose characteristics are known a priori.

Define an error signal as  $e(t) = r(t) + x(t) - s(t)$ . Using the previous definitions, this becomes

$$e(t) = \sum_{i=1}^M a_i u(t-T_i) + \sum_{i=0}^N g_i u(t-iT) + n(t) \quad (1)$$

Since  $e(t)$  is a continuous function of time, some functional will be required to define "optimum" performance. An appropriate performance measure is the  $L_2$  norm. Let  $J_t(g)$  be the cost function used for the time domain analysis.

$$J_t(g) = \int e^*(t)e(t) dt \quad (2)$$

This performance function is also known as the minimum integral square error function (minimum ISE criteria). The problem is to determine  $g = [g_1, g_2, \dots, g_N]^t$  such that  $J_t(g)$  is minimized.

The frequency domain approach is evaluated by first taking the Fourier transform of  $e(t)$ .

$$E(\omega) = \sum_{i=1}^M a_i U(\omega) e^{-jT_i \omega} + \sum_{i=0}^N g_i U(\omega) e^{-j iT \omega} + N(\omega) \quad (3)$$

Again using the  $L_2$  norm

$$J_{\omega}(g) = \int_0^{\infty} E^*(\omega)E(\omega) d\omega \quad (4)$$

#### OPTIMAL WEIGHTS FOR THE TIME DOMAIN MINIMUM ISE

The error signal to be minimized was given to be

$$e(t) = a^t u_r(t) + g^t u_d(t) + n(t) \quad (5)$$

where

$$a = [a_1, a_2, \dots, a_M]^t \quad (6)$$

$$u_r(t) = [u(t-T_{r1}), u(t-T_{r2}), \dots, u(t-T_{rM})]^t \quad (7)$$

$$g = [g_1, g_2, \dots, g_n]^t \quad (8)$$

$$u_d(t) = [u(t), u(t-T), \dots, u(t-NT)]^t \quad (9)$$

The performance function to be minimized is given as

$$J = \int e^*(t)e(t) dt \quad (10)$$

Substituting (5) through (9) and dropping the explicit dependence on  $t$

$$J = \int [a^{*t}u_r + g^{*t}u_d + n] [a^{*t}u_r + g^{*t}u_d + n] dt \quad (11)$$

$$= \int [a^{*t}u_r^{*t}a^{*t}u_r + g^{*t}u_d^{*t}a^{*t}u_r + n^{*t}a^{*t}u_r \quad (12)$$

$$+ a^{*t}u_r^{*t}g^{*t}u_d + g^{*t}u_d^{*t}g^{*t}u_d + n^{*t}g^{*t}u_d$$

$$+ a^{*t}u_r^{*t}n + g^{*t}u_d^{*t}n + n^{*t}n] dt$$

$$= \int [a^{*t}u_r^{*t}u_r^{*t}a + g^{*t}u_d^{*t}u_r^{*t}a + n^{*t}u_r^{*t}a \quad (13)$$

$$+ a^{*t}u_r^{*t}u_r^{*t}g + g^{*t}u_d^{*t}u_d^{*t}g + n^{*t}u_d^{*t}g$$

$$+ a^{*t}u_r^{*t}n + g^{*t}u_d^{*t}n + n^{*t}n] dt$$

Assuming that the noise is uncorrelated with  $u(t)$ , this reduces to

$$= a^{*t}M_{rr} + g^{*t}M_{dr}a + a^{*t}M_{rd}g + g^{*t}M_{dd}g + \sigma_n^2 \quad (14)$$

where

$$M_{rr} = \int u_r^* u_r^t dt \quad (15)$$

$$M_{rd} = \int u_r^* u_d^t dt \quad (16)$$

$$M_{dr} = \int u_d^* u_r^t dt \quad (17)$$

$$M_{dd} = \int u_d^* u_d^t dt \quad (18)$$

Normally a necessary condition for  $g_{opt}$  to minimize  $J$  is that  $\partial J / \partial g^*|_{g_{opt}} = 0$ . However  $g$  is a complex variable and  $J$  is not analytic and the derivative is not defined. Instead define a new variable  $z$  consisting of the real and imaginary parts of  $g$ .

$$g = \sigma_g + j\omega_g \quad (19)$$

$$z = [\sigma_{g1}, \sigma_{g2}, \dots, \sigma_{gN}, \omega_{g1}, \omega_{g2}, \dots, \omega_{gN}]^t \quad (20)$$

Now the necessary condition for  $z_{opt}$  to minimize  $J$  is that  $\partial J / \partial z_i' z_{opt} = 0$ . Taking the partial derivative of  $J$  with respect to  $z$  produces

$$\frac{\partial J}{\partial z} = \frac{\partial g^{*t}}{\partial z} M_{dr}^a + \frac{\partial g^{t \quad t \quad *}}{\partial z} M_{rd}^a + \frac{\partial g^{*t}}{\partial z} M_{dd}^g + \frac{\partial g^{t \quad t \quad *}}{\partial z} M_{dd}^g \quad (21)$$

Noting that

$$\frac{\partial g^t}{\partial z} = \begin{bmatrix} I \\ jI \end{bmatrix} \quad (22)$$

$$\frac{\partial g^{*t}}{\partial z} = \begin{bmatrix} I \\ -jI \end{bmatrix} \quad (23)$$

$$M_{jk}^t = M_{kj}^*, \text{ where } jk \text{ is } rd \text{ or } dd \quad (24)$$

Equation (21) becomes

$$\frac{\partial J}{\partial z} = \begin{bmatrix} I \\ -jI \end{bmatrix} M_{dr} a + \begin{bmatrix} I \\ jI \end{bmatrix} M_{rd} a + \begin{bmatrix} I \\ -jI \end{bmatrix} M_{dd} g + \begin{bmatrix} I \\ jI \end{bmatrix} M_{dd} g^* \quad (25)$$

$$= \begin{bmatrix} M_{dr} a + M_{dr}^* a \\ -jM_{dr} a + jM_{dr}^* a \end{bmatrix} + \begin{bmatrix} M_{dd} g + M_{dd}^* g \\ -jM_{dd} g + jM_{dd}^* g \end{bmatrix} \quad (26)$$

$$= 2 \begin{bmatrix} \text{Re}(M_{dr} a) & \text{Re}(M_{dd} g) \\ \text{Im}(M_{dr} a) & \text{Im}(M_{dd} g) \end{bmatrix} \quad (27)$$

Using (19)

$$M_{dd} g = [\text{Re}(M_{dd}) + j\text{Im}(M_{dd})][\sigma_g + j\omega_g] \quad (28)$$

$$= [\text{Re}(M_{dd})\sigma_g - \text{Im}(M_{dd})\omega_g] + j[\text{Im}(M_{dd})\sigma_g + \text{Re}(M_{dd})\omega_g] \quad (29)$$

Therefore the optimum values of the real and imaginary parts of  $g$  are given by

$$\begin{bmatrix} \text{Re}(M_{dd} g_{opt}) \\ \text{Im}(M_{dd} g_{opt}) \end{bmatrix} = \begin{bmatrix} \text{Re}(M_{dd}) & -\text{Im}(M_{dd}) \\ \text{Im}(M_{dd}) & \text{Re}(M_{dd}) \end{bmatrix} \begin{bmatrix} \sigma_{gopt} \\ \omega_{gopt} \end{bmatrix} \quad (30)$$

$$= \begin{bmatrix} -\text{Re}(M_{dr} a) \\ -\text{Im}(M_{dr} a) \end{bmatrix} \quad (31)$$

$$\begin{bmatrix} \sigma_{gopt} \\ \omega_{gopt} \end{bmatrix} = \begin{bmatrix} \text{Re}(M_{dd}) & -\text{Im}(M_{dd}) \\ \text{Im}(M_{dd}) & \text{Re}(M_{dd}) \end{bmatrix}^{-1} \begin{bmatrix} -\text{Re}(M_{dr} a) \\ -\text{Im}(M_{dr} a) \end{bmatrix} \quad (32)$$

It can be easily shown that (32) can be rewritten strictly in terms of  $g$ ,  $M_{dd}$  and  $M_{dr}$ . From (30) and (31)

$$\operatorname{Re}(M_{dd}g_{\text{opt}}) = -\operatorname{Re}(M_{dr}a) \quad (33)$$

$$\operatorname{Im}(M_{dd}g_{\text{opt}}) = -\operatorname{Im}(M_{dr}a) \quad (34)$$

Which implies

$$M_{dd}g_{\text{opt}} = -M_{dr}a \quad (35)$$

$$g_{\text{opt}} = -M_{dd}^{-1}M_{dr}a \quad (36)$$

This result can now be used to determine the minimum ISE. Substituting (36) into (14)

$$J_{\text{opt}} = a \begin{matrix} *t \\ rr \end{matrix} M \begin{matrix} a \\ + \end{matrix} g \begin{matrix} *t \\ dr \end{matrix} M \begin{matrix} a \\ + \end{matrix} a \begin{matrix} *t \\ rd \end{matrix} M \begin{matrix} g \\ + \end{matrix} g \begin{matrix} *t \\ dd \end{matrix} M \begin{matrix} g \\ + \end{matrix} \sigma_n^2 \quad (37)$$

$$= a \begin{matrix} *t \\ rr \end{matrix} M \begin{matrix} a \\ - \end{matrix} a \begin{matrix} *t & *t & *-t \\ rd & dd & dr \end{matrix} M \begin{matrix} a \\ - \end{matrix} a \begin{matrix} *t & -1 \\ rd & dd & dr \end{matrix} M \begin{matrix} a \\ - \end{matrix} \sigma_n^2 \quad (38)$$

$$+ a \begin{bmatrix} *t & *t & *-t & -1 \\ M & M & M & M & M \\ dr & dd & dd & dd & dr \end{bmatrix} a + \sigma_n^2$$

$$= a^{*t} \begin{bmatrix} M_{rr} & -M_{rd} & M_{dd} & M_{dr} \end{bmatrix}^{-1} a + \sigma_n^2 \quad (39)$$

An improvement factor can now be defined as the ratio of J with and without the adaptive cancellation network. The optimal ISE is given by (39). The ISE without the cancellation network is given by (14) when  $g = 0$  (i.e.  $J = a^{*t} M_{rr} a$ ).

APPENDIX B

DERIVATION OF CONTROL EQUATIONS  
BASED ON AN ITERATIVE LEAST MEAN  
SQUARES GRADIENT APPROACH

The optimal weight for the adaptive nulling network,  $g_{opt}$ , was given in (36) as  $-M_{dd}^{-1}M_{dr}a$ . While this provides a closed form solution, its direct implementation causes some problems. First, the matrices  $M_{dd}$  and  $M_{dr}$  may not be available. Second, the inverse may be difficult to compute in real time. Third, good nulling performance requires a high degree of computational accuracy. An alternate implementation makes use of the gradient method to iteratively approach the solution given by (36).

Let  $\nabla_g J = \partial J / \partial g$  define the gradient of  $J$  with respect to  $g$ . Then the gradient algorithm (which is a method of steepest decent) is given by

$$g(n+1) = g(n) - k \nabla_g J(n) \quad (1)$$

where  $k$  is a convergence factor. An estimate of the gradient is obtained using readily available signals from the adaptive nulling processor.

An expression for  $\partial J / \partial z$  was given in (27) where  $z = [\sigma_g^t, \omega_g^t]^t$ . A complex representation is obtained by noting that the upper and lower terms in each matrix represents the real and imaginary parts respectively. Therefore, the complex form of the gradient is given by

$$\frac{\partial J}{\partial g} = M_{dr} a + M_{dd} g \quad (2)$$

$$= \int_d^* u u a \tau dt + \int_d^* u u g \tau dt \quad (3)$$

$$= \int_d^* u r dt + \int_d^* u x dt \quad (4)$$

$$= \int_d^* u (r + x) dt \quad (5)$$

$$= \int_d^* u (e + s) dt \quad (6)$$

$$= \int_d^* u e dt \quad (7)$$

A discrete approximation is given by the recursive relation

$$\nabla_g J(n) = \nabla_g J(n-1) + T u_d^*(n) e(n) \quad (8)$$

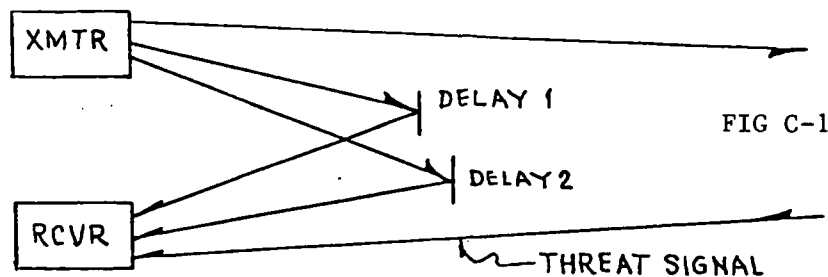
where T is the time between samples.

APPENDIX C

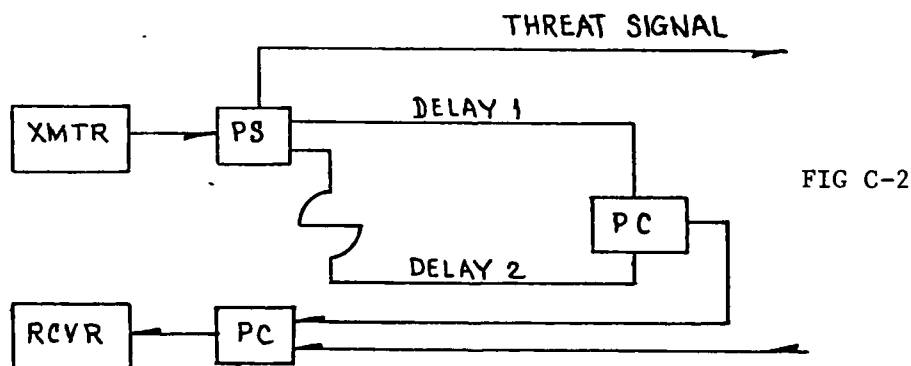
THE ECM LOOKTHROUGH MULTIPATH  
REFLECTION ENVIRONMENT

In a system that must receive and transmit simultaneously, signals from the transmitter find their way into the receiver and raise the receiver noise level. These signals may reflect from nearby structure, or may couple directly by way of antenna sidelobes. Cancellation of either mode can be accomplished if the sample of the transmitted signal is coherent, or can be made coherent with, the received signal. In this section, time, frequency and phase domain descriptions of the problem and solutions will be presented.

C.1 A model of the reflected case is illustrated in Figure C-1



A model of the reflection model may be simulated with delay lines as shown in Figure C-2.



The delay 1 and 2 signals are summed with received signals to produce higher noise levels.

C.2 To null an interfering signal, the feedback signal must be controlled in phase ( $\phi$ ), gain (G) and delay (D). It is useful to discuss the problem in time, phase as a function of frequency, and amplitude as a function of frequency.

Signals, separated by a few nanoseconds may arrive at the receiver to interfere with other incoming signals. This situation is illustrated in the time domain in Figure C-3.

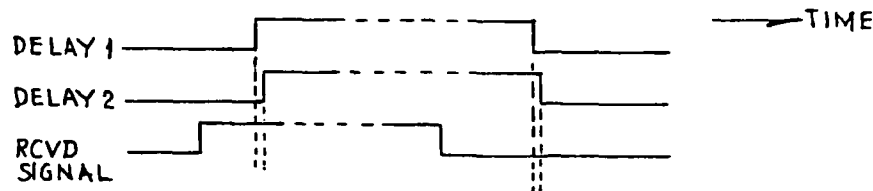


FIG C-3

C.3 A delay line has phase as well as time characteristics. The phase characteristic of a delay line shows linearly increasing phase as a function of frequency. (See Figure C-4)

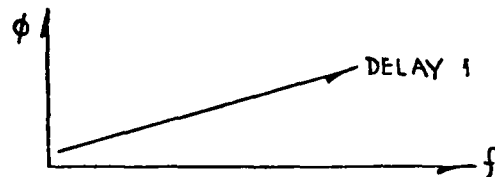


FIG C-4

The slope of the phase/frequency characteristic is controlled by the length of the cables. Two cable characteristics are illustrated in Figure C-5 as Delay 1 and 2.

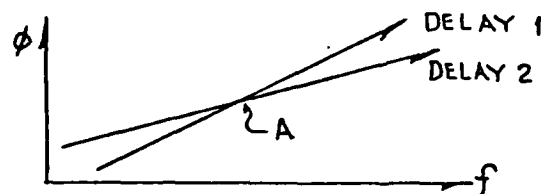


FIG C-5

If the signals from Delay 1 and 2 were subtracted, there would be a null at Point A. (assuming the amplitude of A equals the amplitude of B).

C.4 In the frequency domain, the addition of the signals shown in Figure C-5 would produce a multi-cusp amplitude frequency characteristic as shown in Figure C-6.

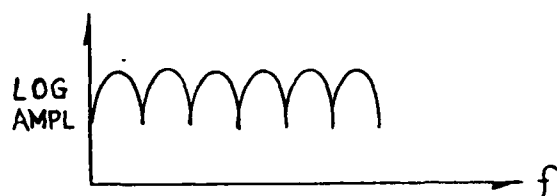


FIG C-6

The cusps are formed at points when phase equals 0, 360, 720 degrees. The distance between cusps increases as the differential length of the cables decreases. Cables of equal length produce no cusps.

#### C.5 How to form a null - single reflection

There are two ways to null a single reflection. A delayed signal equal to the reflected signal in phase, gain, and delay will be called the 1-on-1 strategy. The 2-on-1 strategy is formed by setting the delays of 2 feedback channels unequal to the reflected signal. The phase and gain are set for maximum bandwidth null. The 2-on-1 strategy is the simplest case of N-on-1.

##### C.5.1 1-on-1 Null

If a single uncontrolled delay (reflected path) is matched with a path that is controlled in delay, phase and amplitude, a null of broadband may be formed. Figure C-7 illustrates such a circuit.

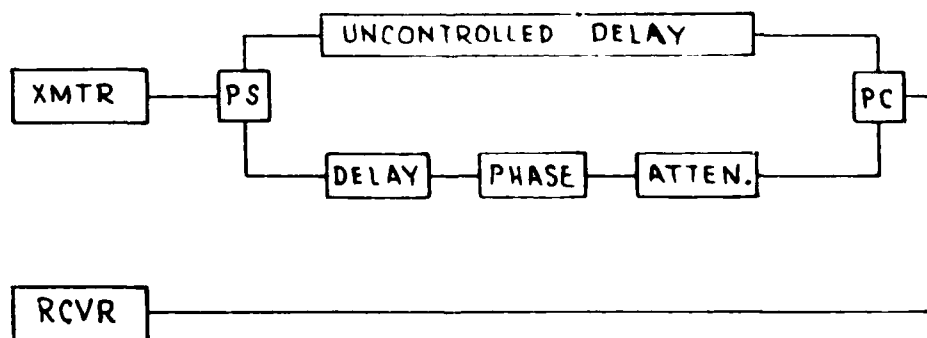


FIG C-7

If phase, gain and delay can be matched between the two channels from  $f_0$  to  $f_1$  a null will be formed. If phase or gain are not equal outside of  $f_0$  and  $f_1$ , the null quality suffers. Figure C-8 shows the phase characteristic. Figure C-9 shows the amplitude null characteristic.

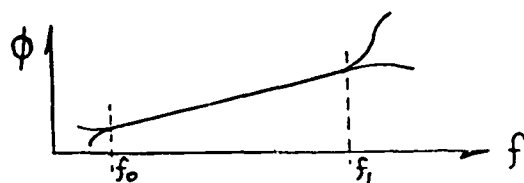


FIG C-8

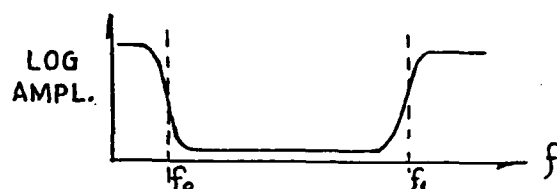


FIG C-9

#### C.5.2 2-on-1 Null

A 2-on-1 null is produced by combining 2 controlled phase/gain paths with one uncontrolled (reflected) path. A simplified Block Diagram is shown as Figure C-10.

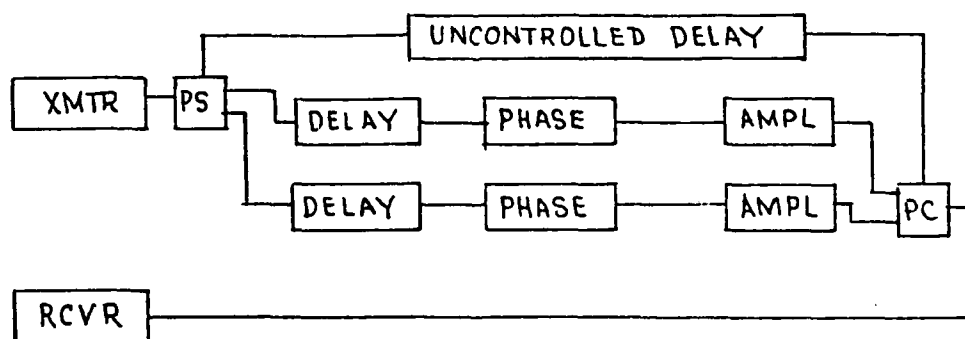


FIG C-10

In this method, the delays are not matched; instead they are set to bracket the uncontrolled pulse. This relationship is shown in Figure C-11.  $\tau$  might equal 4 nsec.

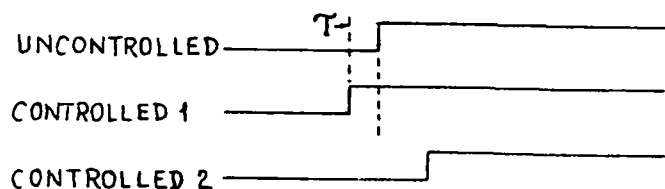


FIG C-11

The value of  $\tau$  is set to produce the wanted bandwidth of null. The equation relating null bandwidth to  $\tau$  is  $BW = .4/\tau$ .

C.5.2.1 The 2-on-1 strategy is illustrated, in the phase domain, in Figure C-12

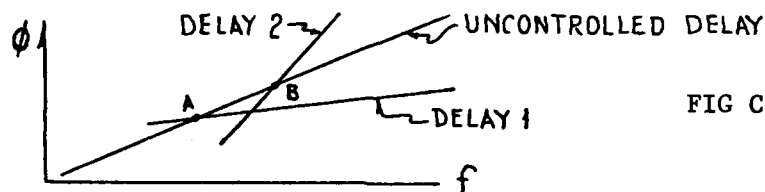


FIG C-12

The intersections (A and B) represent null points. The combination of 2 phase slopes produces a characteristic of a different slope. Figure C-13 illustrates the phase slopes of 2 delay lines.

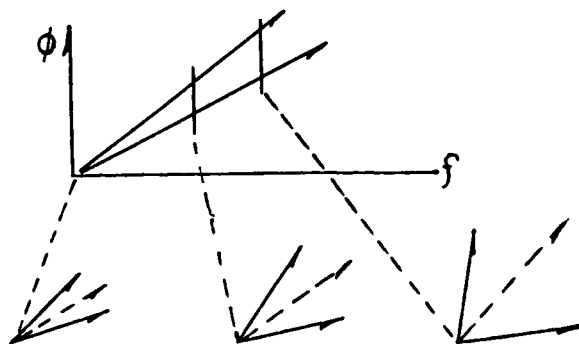


FIG C-13

At three points on 2 phase slopes, vector diagrams represent the phase relationship of each curve and the resultants. It is apparent that the resultant slope is intermediate between the two original slopes. With reference to Figure C-12, the two controlled delay slopes add to match the uncontrolled delay slope from A to B and beyond.

#### C.5.2.2 Frequency Domain Illustration of 1-on-1 Strategy

The frequency domain representation of a 2-on-1 strategy can be viewed as 2 pairs of delays. One pair is made from the uncontrolled path plus delay 1; the other is an uncontrolled path plus delay 2. The amplitude/frequency plots are shown as Figures C-14 and C-15. The cusps A and B are related to crossing points A and B in Figure C-12.

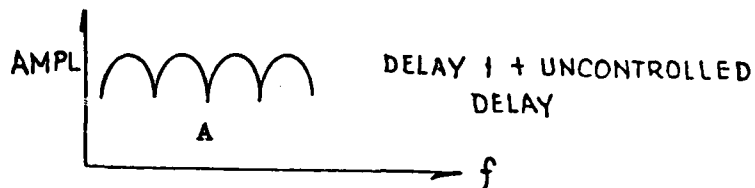


FIG C-14

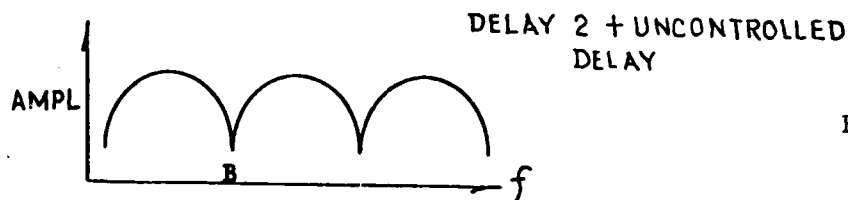


FIG C-15

The cusps can be controlled by changing phase, as shown in Figure C-16.

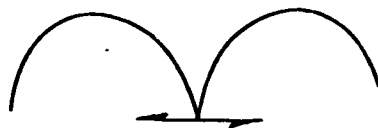


FIG C-15

The cusps shown in Figures C-14 and C-15 can be moved in frequency so that they are coincident as shown in Figure C-17. The cusps would be set

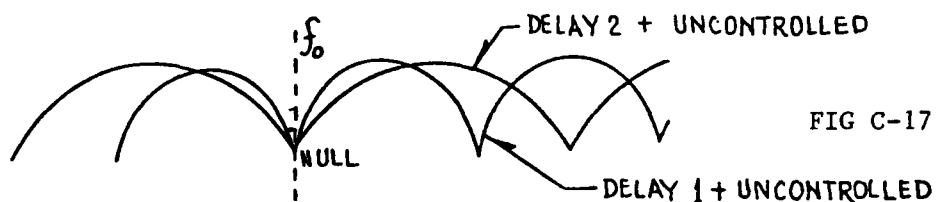


FIG C-17

with a slight offset to produce the situation of Figure C-12 and C-18.

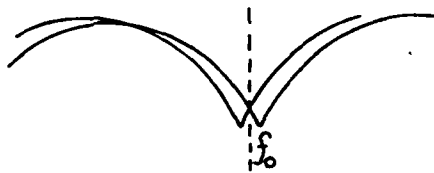


FIG C-18

#### C.6 Channel Delays in the Time Domain

A reflecting surface with steps in range will produce delayed pulses with steps in time. A reflected pulse with multiple delays would have a form as shown in Figure C-19.

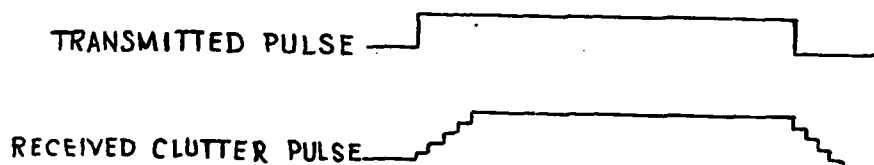


FIG C-19

If the transmitted pulse were delayed to match the received pulse with a single delay, the difference between the pulses would contain "dog ears". Figure C-20 shows this relationship. The step time of the resultant video waveform might be in the order of 4 nsec.

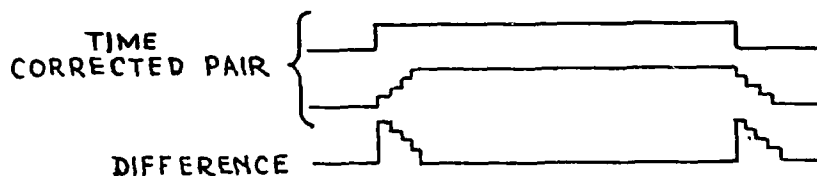
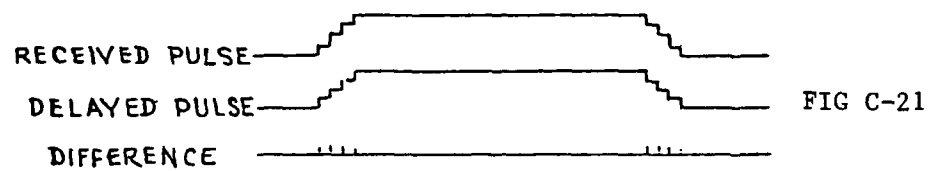


FIG C-20

These "dog ears" will degrade the null somewhat, and might produce unwanted video noise, depending on the system.

The ideal time relationship between reflected and feed back signal is shown in Figure C-21. The stairsteps are equal and a good video null is formed.



APPENDIX D

CONTINUOUS SIGNAL MODEL  
OF CANCELLATION PERFORMANCE

The following paragraphs establish simple equation models for the cancellation of a continuous wave signal with very slow modulation using one cancellation tap or two taps. These models are used for comparison with simulation results in the main section of the report.

Define a cw signal of unit amplitude which is changing slowly in frequency  $f$  over a frequency band of width  $B$  MHz.

$$s(t) = e^{j2\pi ft}$$

(eqn D - 1)

A copy of the signal is delayed by  $T$  and combined with the original signal such that cancellation would be complete at the center of the band  $f_0$ .

$$\begin{aligned} c(t) &= e^{j2\pi ft} - e^{j2\pi f(t + \Delta T)} + \cancel{0} \\ &= e^{j2\pi ft} (1 - e^{j2\pi f\Delta T} + \cancel{0}) \end{aligned}$$

(eqn D - 2)

At  $f = f_0$ ,

$$c(t) = 0$$

(eqn D - 3)

$$e^{j2\pi f\Delta T + \phi} = 1$$

(eqn D - 4)

Taking first solution

$$\phi = -2\pi f_0 \Delta T$$

(eqn D - 5)

Rewriting equation 2,

$$c(t) = (1 - e^{j2\pi(f-f_0)\Delta T})e^{j2\pi ft}$$

(eqn D - 6)

The magnitude squared of the combined signal is

$$\begin{aligned} c_m(t) &= |c(t)|^2 = \left| 1 - e^{j2\pi(f-f_0)\Delta T} \right|^2 \\ &= (1 - \cos(2\pi(f-f_0)\Delta T))^2 + \sin^2(2\pi(f-f_0)\Delta T) \end{aligned}$$

$$= 2 - 2\cos(2\pi(f-f_0)\Delta T) \quad \text{(eqn D - 7)}$$

The logarithmic signal cancellation level relative to the original signal is defined as

$$C = 10 \log_{10} (1/ cm(t) )$$

(eqn D - 8)

At band edge ( $f = f_o + B/2$ ), the cancellation ratio for one tap is

$$C_1 = 10 \log_{10}(1/(2 - 2 \cos(\pi B \Delta T)))$$

(eqn D - 9)

Averaged over the band ( $f_o - B/2, f_o + B/2$ ), the cancellation ratio for one tap is

$$\begin{aligned} C_2 &= 10 \log_{10} \left[ \frac{1}{\frac{1}{B} \int_{f_o - B/2}^{f_o + B/2} (2 - 2 \cos(2\pi(f - f_o)\Delta T)) df} \right] \\ &= 10 \log_{10} \left[ \frac{1}{2 - 2 \sin(\pi B \Delta T) / \pi B \Delta T} \right] \\ &\approx 10 \log_{10} (3/\pi^2 (B \Delta T)^2) \end{aligned}$$

(eqn D - 10)

The above equations model performance of a single tap cancellation for a simple continuous signal.

Two half amplitude copies of the signal  $s(t)$  can be timed  $\Delta T/2$  in advance of  $s(t)$  and  $\Delta T/2$  later in time than  $s(t)$ . The magnitude of the combined signal is

$$cm(t) = \left| 1 - 0.5e^{j2\pi (f-f_o)\Delta T/2} - 0.5e^{j2\pi (f-f_o)\Delta T/2} \right|^2$$

$$= (1 - \cos(\pi (f-f_o)\Delta T))^2$$

(eqn D - 11)

The logarithmic signal cancellation level relative to the original signal is

$$C = 10 \log_{10} (1 / (1 - \cos(\pi (f-f_o)\Delta T)))^2$$

(eqn D - 12)

At band edge ( $f = f_o + B/2$ ), the cancellation ratio for two taps is

$$C_3 = 10 \log_{10} (1 / (1 - \cos(\pi B\Delta T/2)))^2$$

(eqn D - 13)

Averaged over the band ( $f_o - B/2, f_o + B/2$ ), the cancellation ratio for two taps is

$$c_4 = 10 \log_{10} \left[ \frac{1}{\frac{1}{B} \int_{f_0 - B/2}^{f_0 + B/2} (1 - \cos(\pi(f-f_0)\Delta T))^2 df} \right] \quad (\text{eqn D - 14})$$

where

$$\begin{aligned} & \frac{1}{B} \int_{f_0 - B/2}^{f_0 + B/2} (1 - \cos(\pi(f-f_0)\Delta T))^2 df = \\ & = 1 + \frac{1}{2} - \frac{4}{B\Delta T\pi} \sin(\pi B\Delta T/2) + \frac{1}{2\pi B\Delta T} \sin(\pi B\Delta T) \end{aligned} \quad (\text{eqn D - 15})$$

$C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$  are plotted in figure 2-2 of the main section of the report for varying  $B$   $T$ .

APPENDIX E

DETAILED DESCRIPTION OF SIMULATIONS

1.     INTRODUCTION

The simulation includes five distinct models which apply different solutions to the cancellation problem, one of which uses the weight vector found with a reference waveform and tests it against another waveform. These models are as follows:

MST1     Sweep mode which provides a sequence of successive single-point cancellations. Multiple odd and even sweeps over interlaced sets of delay line taps is an option.

MST3     Simultaneous equation calculation for exact cancellation at times matching n taps.

MST4     Matrix inversion approximation to LMS criterion.

MSTGR    Gradient (steepest descent) solution to LMS criterion (iterative).

MSTX     Test of weight vector generated by one of the other models using a different reference waveform.

A variation of MST4 which takes advantage of the real-only reference waveforms to reduce storage requirements and running time is designated MST4RL. This has been obtained from MST4 by adding by-passes, etc., rather than deletions, and therefore contains all the coding of MST4. It will be used for detailed discussion and supplemented as necessary. Each of these models conforms to Figure E-1. The models are supported by a separate program, FILT, which provides the reference waveform for a specified filter and driving signal, representing the effect of all band-limiting filtering in the system. This previously existing program will be briefly reviewed; the mathematical basis is detailed in Appendix F.

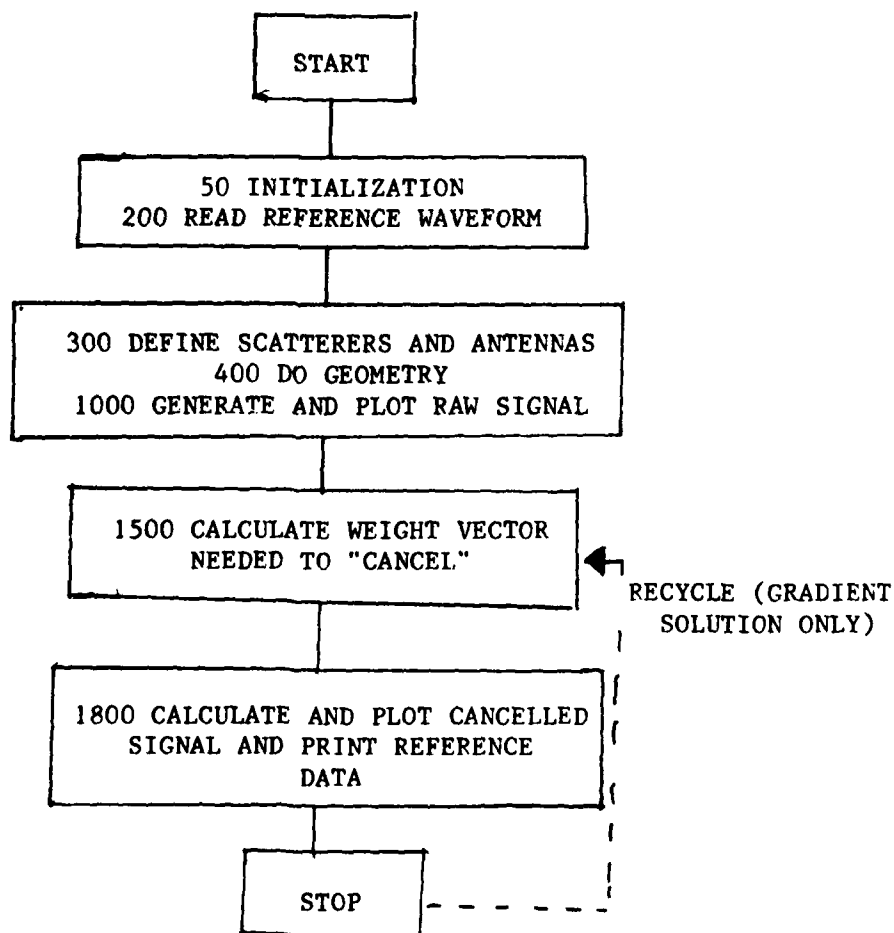


FIGURE E-1

The MST1 and MST3 models were initially implemented on an HP-9830. For speed and storage, the HP-9830 was replaced by a small computer of the S-100 variety. On the S-100 computer the 9830 matrix package had to be replaced by interpreter code (lines 3000-ff).

In what follows, the term "vector" always means a matrix of size  $n \times 1$  for some value of  $n$ , never a complex number  $a + jb$ , such as a phasor. The elements of a matrix are themselves usually complex numbers. A complex matrix will be denoted as  $[A + jB]$  where  $A$  and  $B$  are real matrices, usually with no explicit representation of size or subscripts. When subscripted, the matrix brackets are omitted.

## 2. DETAILED DESCRIPTION OF MST4 AND MST4RL: MATRIX/LMS SOLUTION

Figure E-2 is a map of model MST4 and MST4RL. The distinction between MST4 and MST4RL lies in restriction of MST4RL to real reference waveforms, indicated by enclosing the identically zero imaginary parts  $[jS]$ ,  $[jQQ]$ ,  $[jBB\#]$ , and  $[jDD\#]$  in parentheses in the Names column. (Note the brackets here since these are in fact matrices.) This chart is equivalent to a flow diagram. It is presented in line number sequence with jump arrows as necessary and includes subroutines and patches, whose position in the sequence arose from a process of evolution. Discussion of self-explanatory matters will be omitted and incidental items will be defined. The complete listing of MST4RL appears in Listing E-1. Table E-1 supplements the list of selected variable names shown in the Names column of Figure E-2.

FIGURE E-2

| SECTION  | LINE | PROCESS(ES)  | NAMES        | SUB/PATCH                                   |
|--|------|--|--------------|---|
| Preliminary  | 50   | Initializations,<br>declarations, header, etc                        | [R(+jS)]     |   |
|  | 200  | Read and Store reference<br>waveform                                 |              |   |
| Signal   | 300  | Initialize RN generator  |              |   |
|  | 310  | Input scatterer and<br>antenna parameters                            |              |   |
|  | 400  | Do geometry (delay.<br>phase); get min. delay                        |              | Sub 2000                                    |
|  | 1000 | Calculate raw signal at<br>each time<br>Print scatter times          | [B+jC]       | Sub 2000<br>Sub 2600                        |
| Patch  | 1100 | Scan for mean and worst<br>print data; STOP                          |              |   |
|  | 1200 | Store raw signal (working<br>copy to be cancelled<br>Plot raw signal | [U+jV]       | Sub 3900<br>Sub 1350<br>(alt.1400,<br>1480) |
| Subroutine   | 1350 | Plot a curve on printer  |              |   |
| Subroutine   | 1400 | Do scaling and axes on<br>plotter                                    |              |   |
| Subroutine   | 1480 | Plot a curve on plotter  |              |   |
| Cancellation<br>(calculation<br>of weight<br>vector) | 1500 | Input cancel parameters  |              |   |
|  | 1550 | Set up delay line output<br>[U]<br>Print it                          | [pp(+j00)]   | Patch 4200                                  |
|  |      | Calculate $[u^*T][u]$ and<br>print it                                | [AA#(+jBB#)] | Patch 3000                                  |
|  |      | Invert it  | [CC#(+jDD#)] | Sub 6000<br>(patch 3400<br>real only)       |
|  | 1600 | Set up "target vector" (raw<br>signal at cancel times)               | [FF+jGG]     |   |
|  | 1650 | Calculate $[u^*T]$ (target<br>vector) and print it                   | [X+jYY]      | Patch 4000                                  |
|  | 1660 | Calculate weight vector<br>and print it                              | [WW+jZZ]     |   |

| SECTION             | LINE | PROCESS(ES)   | NAMES  | SUB/PATCH                            |
|---------------------|------|---|--------|--------------------------------------|
| Cancelled<br>signal | 1800 | Calculate cancelled signal<br>Plot it                               | [U+jV] | Sub 1350<br>(alt 1480)<br>Patch 1100 |
|                     |      | Print results and reference<br>data; STOP                           |        |                                      |
| Subroutine          | 2000 | Calculate raw signal at a time<br>point (delay only if KEY = 0)     |        |                                      |
| Subroutine          | 2600 | Print scatter times   |        |                                      |
| Subroutine          | 2800 | Plot a point on printer   |        |                                      |
| Patch               | 3000 | Calculate $[U^*T]$ [u]  |        |                                      |
| Patch               | 3400 | Store real matrix as vector<br>for inversion and recover<br>inverse |        | Sub 6400                             |
| Subroutine          | 3900 | Store working copy of signal  |        |                                      |
| Patch               | 4000 | Calculate $[u^*T]$ (target vector)                                  |        |                                      |
| Patch               | 4200 | Print delay line output matrix                                      |        |                                      |
| Subroutine          | 6000 | Invert complex matrix   |        | Sub 6400                             |
| Subroutine          | 6300 | Matrix product (special case<br>repeated)                           |        |                                      |
| Subroutine          | 6400 | Invert a real matrix stored as<br>a vector                          |        |                                      |

TABLE E-1

|                 |   |
|-----------------|---|
| T               | t for stored reference waveform                             |
| T1              | t for signal waves  |
| C               | velocity  |
| NSK             | skip factor = $T1/T$  |
| JCP             | key point of ref. waveform (approx. peak)                   |
| NSC,D ,G ,D9    | scatterers (number, range, bearing, spread)                 |
| X ,Y            | coordinates of scatterers group                             |
| F7,S7,NLS,X4,   | frequency, element spacing ( ), no. of                      |
| Y4              | elements  |
| X4,Y4           | receive antenna coordinates                                 |
| WNR,S2          | wave number, element spacing                                |
| X(I),Y(I),Z(I)  | scatterer coordinates, strength                             |
| D(I,J),E(I)     | delays: element J to scatterer I,<br>scatterer I to receive |
| F(I,J),G(I)     | Phases: corresponding to delays<br>principal value)         |
| DS,WX,AMAX      | shortest delay, signal amplitude, max.<br>amplitude         |
| LPS,LPE,LPD,MP  | cancel time points: start, end, step,<br>number             |
| LTS,LTE,LTD,NCL | taps (identified by time): start, end<br>step, number       |
| UU#(I,J),       | storage needed in complex matrix                            |
| VV#(I,J)        | inversion   |
| W#(K)           | Vector storage of matrix for inversion                      |
| LL(K),MM(K)     | storage needed in real matrix inversion                     |

LISTING E-1

COMPLETE LISTING OF MST4RL/MST4 -- DIRECT MATRIX INVERSION/LMS SOLUTION

```

1  '+++++++ THIS IS THE START--FILE BEGINS WITH "1 '+++..."ETC.
11 'MODEL OF MULTIPLE REFLECTIONS
12 '   GENERATES RAW REFLECTION SIGNAL
13 '   MATRIX INVERSION SOLUTION TO LMS CANCELLATION
15 '   REAL WAVEFORM (SUBSTITUTE 200-280 A/R)
16 '   UNNEEDED COMPLEX CALCS BY-PASSED
20 '   LINES BY-PASSED, SUBSTITUTED:
21 '       1510,1512
22 '       1520,1522
23 '       1532,1534
24 '       1594,1596
25 '       1680,1682
26 '       1690,1692
27 '       1850,1852
28 '       1860,1862
29 '       2400,2402
30 '       3120,3122
31 '       3130,3132
32 '       4120,4122
33 '   LINES CUT SHORT:
34 '       1570
35 '       1580
36 '       2320
37 '       3110
38 '       4110
39 '   LINES BY-PASSED
40 '       3170
41 '       3180
42 '       3190
43 '       4230
44 '       4240
50 GOTO 84 ' BY-PASS WHEN THERE'S NO MEMORY SPACE
52 '   INSERT PLOTTER INITIALIZATION INTO MEMORY
60 POKE &HF6D7,&HC3:POKE &HF6D8,0:POKE &HF6D9,&HFE
62 POKE &HFE00,&H3A:POKE &HFE01,&H1C:POKE &HFE02,&HFE
64 POKE &HFE03,&HB7:POKE &HFE04,&H28:POKE &HFE05,&H6
66 POKE &HFE06,&H3A:POKE &HFE07,3:POKE &HFE08,0
68 POKE &HFE09,&HC3:POKE &HFE0A,&HDA:POKE &HFE0B,&HF6
70 POKE &HFE0C,&HCD:POKE &HFE0D,6:POKE &HFE0E,&HF6
72 POKE &HFE0F,&HC0:POKE &HFE10,0:POKE &HFE11,0
74 POKE &HFE12,&HDB:POKE &HFE13,&HFB:POKE &HFE14,&HE6
76 POKE &HFE15,1:POKE &HFE16,&H28:POKE &HFE17,&HF4
78 POKE &HFE18,&H79:POKE &HFE19,&HD3:POKE &HFE1A,&HFB
80 POKE &HFE1B,&HC9
82 POKE &HFE1C,0'-----FLAG 0 TO PLOT,1 TO LPRINT
84 'INVERSE OF ABOVE
86 'POKE &HF6D7,58:'POKE&HF6D8,3:'POKE&HF6D9,0
88 'POKE 65052,1 'MAKE SURE NOT TO DRIVE PLOTTER YET
90 DEFINT I-N
94 INPUT "DATE NR.,TIME,DISK ID";IDATE,ITIME,DSKS:LPRINT CHR$(12)
96 LPRINT CHR$(10);CHR$(10)
98 LPRINT "START OF RUN";IDATE;ITIME;"   MST4RL   DISK ";DSKS:LPI

```

```

100 DIM R(51),S(51)
120 DIM U(61),V(61)
130 DIM B(61),C(61)
140 DIM PLT$(80)
150 DEF FNF(X)=X-INT(X)
152 DEF FNX(X)=INT(X6*(X-X5))
154 DEF FNY(Y)=INT(Y6*(Y-Y5))
160 PI#=3.14159265#:PIO2#=PI#/2#:PI2#=2#*PI#:DTR#=PI#/180#:C0#=.3#
169 '-----CHARACTERS AND ARRAY FOR PRINTER PLOT
170 LB$="*":AST$="*":BLK$=" ":PDS$="."
172 FOR I=1 TO 80:PLT$(I)=BLK$:NEXT I
174 FOR I=5 TO 80 STEP 10:PLT$(I)=PDS$:NEXT I
190 FACT=1:NTS=61
199 '-----WAVEFORM AS TRANSMITTED AND FED TO DELAY LINE
200 FOR I=1 TO 51
210 READ R(I):S(I)=0:NEXT I
220 DATA 0,.033,.13,.261,.404,.542,.667,.774,.86,.926,.974,1.007
230 DATA 1.028,1.039,1.043,1.042,1.038,1.033,1.026,.97,.857
240 DATA .717,.571,.432,.309,.205,.123,.06,.015,-.015,-.033
250 DATA -.042,-.044,-.042,-.038,-.032,-.026,-.019,-.014,-.009
260 DATA -.006,-.003,-.001,0,.001,.002,.002,.002,.002,.001,.001
270 T0=.4:T1=.4:NSK=1:JCP=16:BW=250:PW=7.11
280 LPRINT "BW IS ";BW;" PW IS ";PW;" T0,JCP ARE ";T0;JCP
300 S8=RND(-2+PI#/2)
306 INPUT "RN SLIP";JRN:IF JRN=0 GOTO 310
308 FOR I=1 TO JRN:S8=RND:NEXT I
309 '-----INPUT CASE PARAMETERS
310 INPUT "NR.SCATT'S,GROUP COORD'S,SPREAD";NSC,D0,G0,D9
320 X0=D0*SIN(G0*DTR#):Y0=D0*COS(G0*DTR#)
330 X4=-.25:Y4=0
340 INPUT "FREQ, SPACING (WL), NR. ELEMENTS";F7,S7,NLS
342 VNLS=NLS
344 WNR=F7/C0#:S2=S7/WNR
350 DIM X(NSC),Y(NSC),Z(NSC)
360 DIM D(NSC,NLS),E(NSC),F(NSC,NLS),G(NSC)
390 LPRINT BW;T1;JRN:LPRINT NSC;D0;G0;D9:LPRINT F7;S7;NLS;X4:LPRINT
399 '-----DO GEOMETRY: DELAY, PHASE ON ALL PATHS
400 FOR I=1 TO NSC
410 Z[I]=5:X1=X0+D9*RND:X[I]=X1
430 Y1=Y0+D9*RND:Y[I]=Y1
440 U3=X1-X4:V3=Y1-Y4
460 D3=SQR(U3*U3+V3*V3)
470 E[I]=D3/C0#:G[I]=-FNF(D3*WNR)*PI2#'-----RETURN LEG
500 FOR J=1 TO NLS
502 VJ=J
510 X2=S2*(VJ-.5*(NLS+1)):U2=X1-X2
530 D2=SQR(U2*U2+Y1*Y1)
540 D[I,J]=D2/C0#:F[I,J]=-FNF(D2*WNR)*PI2#'-----OUTGOING LEG
560 NEXT J
690 NEXT I
800 KEY=0'-----GET MIN DELAY AS TIME REF
810 D5=1E+06
820 GOSUB 2000
900 PRINT "GEO DONE -- START RAW SIG CALC VS. TIME"
1000 B(1)=.000001

```

```

1002 C(1)=.000001
1010 AMAX=0
1020 FOR NT=2 TO NTS
1030 KEY=1'-----CALC SIG VIA ALL PATHS AT TIME POINT
1040 GOSUB 2000
1042 WX=SQR(U*U+V*V)'-----SAVE MAX AS REF
1044 IF AMAX>WX THEN 1050
1046 AMAX=WX
1050 B[NT]=U'-----STORE (RECT)
1052 C[NT]=V
1060 NEXT NT
1070 LPRINT:LPRINT D5;AMAX:LPRINT
1074 LPRINT "SCATTER TIMES (KEY POINT)":LPRINT
1080 GOSUB 2600'-----PRINT TIMES
1082 GOTO 1200
1099 '-----CALC AND PRINT SUMMARY DATA AND STOP
1100 SUM=0:VWRST=0:NTA=0:NRST=999
1110 FOR NT=1 TO NTS
1112 U=U(NT):V=V(NT):WX2=U*U+V*V:WX=SQR(WX2)
1114 IF NT<LPS-LPD/2 OR NT>LPE+LPD/2 GOTO 1120
1116 SUM=SUM+WX2:IF VWRST<WX THEN VWRST=WX:NRST=NT
1118 NTA=NTA+1
1120 NEXT NT
1140 DBMN=8.68589*LOG(SQR(SUM/NTA)/AMAX)
1142 DBWRST=8.68589*LOG(VWRST/AMAX):LPRINT
1144 LPRINT "MEAN POWER REL. TO ORIG. MAX (DB) ";DBMN
1146 LPRINT "WORST POINT REL. TO ORIG. MAX (DB) ";DBWRST;"AT PT.
";NRST
1150 LPRINT:LPRINT "BW,DELTA-T,RN SKIP ";BW;T1;JRN
1152 LPRINT "SCATTERERS ";NSC;D0;G0;D9
1154 LPRINT "ANTENNAS ";F7;S7;NLS;X4
1156 LPRINT "TARGET PTS:LPS,LPE,LPD,MP ";LPS;LPE;LPD;MP
1158 LPRINT "TAPS:LTS,LTE,LTD,NCL,JCP,SPCG ";
1159 LPRINT LTS;LTE;LTD;NCL;JCP;LTD*T1
1160 STOP
1200 GOSUB 3900 '(RE)STORE RAW SIG AS WORKING COPY
1210 GOSUB 1350 'TO USE PLOTTER INACTIVATE THIS
1220 GOTO 1500 'AND THIS -- ALSO SWITCH 1890 TO 1892
1250 GOSUB 1400 'DO AXES AND TICK LABELS
1270 GOSUB 1480 'PLOT SIG (RAW OR CANCELLED)
1290 GOTO 1500 'TO CANCEL
1350 ' PLOT ON PRINTER
1352 LPRINT " -30 -20
-10";
1354 LPRINT " 0 DB"
1360 FOR NT=1 TO NTS
1362 U=U(NT):V=V(NT):WX2=U*U+V*V:WX=SQR(WX2)
1364 GOSUB 2800 'PRINT A PLOT LINE
1376 NEXT NT
1394 LPRINT
1398 RETURN
1400 X5=0:X6=25*T1/.4:Y5=-45:Y6=5'-----DO SCALING; DO AXES AND
LABELS
1402 X6=4095/(X6-X5):Y6=4095/(Y6-Y5)
1404 ' POKE 65052,0

```

```

1406 LPRINT "#";FNY(0);INT(X6*2.5*T1/.4)
1408 LPRINT "*" ;FNX(0);INT(Y6*5)
1410 ' DO TICK LABELS -- HERE TO 1476
1478 ' POKE 65052,1
1479 RETURN
1480 ' POKE 65052,0 '-----PLOT A CURVE
1482 FOR NT=1 TO NTS
1484 U=U(NT):V=V(NT):T=(NT-1)*T1
1486 WX=SQR(U*U+V*V)
1488 IF WX/AMAX<.0056235 THEN DB=-45:GOTO 1492
1490 DB=8.6859*LOG(WX/AMAX)
1492 LPRINT "'";FNX(T);FNY(DB)
1494 NEXT NT
1496 LPRINT "$"
1497 'POKE 65052,1
1498 RETURN'++++++THIS IS LINE 1498--"1498 RETURN'+++..."ETC.
1499 '-----BEGIN CANCEL PROCEDURE
1500 INPUT "LPS,LPE,LPD,MP";LPS,LPE,LPD,MP
1504 INPUT "LTS,LTE,LTD,NCL";LTS,LTE,LTD,NCL:NCL2=NCL*NCL
1508 LPRINT LPS;LPE;LPD;MP;" ";LTS;LTE;LTD;NCL;JCP
1510 'DIM AA#(NCL,NCL),BB#(NCL,NCL),CC#(NCL,NCL),DD#(NCL,NCL)
1512 DIM AA#(NCL,NCL),CC#(NCL,NCL)
1520 'DIM UU#(NCL,NCL),VV#(NCL,NCL),XX(NCL),YY(NCL),W#(NCL2)
1522 DIM XX(NCL),YY(NCL),W#(NCL2)
1530 DIM WW(NCL),ZZ(NCL),LL(NCL),MM(NCL)
1532 'DIM PP(MP,NCL),QQ(MP,NCL),FF(MP),GG(MP)
1534 DIM PP(MP,NCL),FF(MP),GG(MP)
1540 LPRINT:LPRINT "INDICES FOR PICK-UP OF R,S FOR MATRIX"
1549 '-----PICK UP DELAY LINE OUTPUT MATRIX
1550 FOR I=1 TO MP
1552 NTI=LPS+(I-1)*LPD
1554 FOR J=1 TO NCL
1556 NTJ=LTS+(J-1)*LTD
1560 I7=(NTI-NTJ)*NSK+JCP
1562 IF I7<1 THEN U7=0:V7=0:GOTO 1580
1564 IF I7>51 THEN U7=0:V7=0:GOTO 1580
1570 U7=R(I7):V7=S(I7)
1580 PP(I,J)=U7:QQ(I,J)=V7
1590 LPRINT I7;:NEXT J:LPRINT:NEXT I
1591 GOTO 4200'-----PRINT THE MATRIX
1592 GOTO 3000'-----GET U(*,T) TIMES U AS AA#+jBB# FOR INVERSION
1594 'GOSUB 6000'-----GET INVERSE AS CC#+jBB#
1596 GOTO 3400'----- (THIS IS FOR REAL-ONLY WAVEFORMS)
1600 LPRINT:LPRINT "TARGET VECTOR (RAW SIGNAL AT CANCEL POINTS)"
1610 FOR I=1 TO MP
1620 I7=LPS+(I-1)*LPD
1630 U7=U(I7):V7=V(I7)
1632 FF(I)=U7:GG(I)=V7
1640 LPRINT I7;U7;V7:NEXT I:LPRINT
1650 GOTO 4000'-----GET U(*,T) TIMES TARGET VECTOR
1660 '-----DO FINAL PRODUCT TO GET WEIGHT VECTOR
1670 FOR I=1 TO NCL:SW=0:SZ=0
1680 'FOR J=1 TO NCL:TC=CC#(I,J):TD=DD#(I,J):TX=XX(J):TY=YY(J)
1682 FOR J=1 TO NCL:TC=CC#(I,J):TX=XX(J):TY=YY(J)
1690 'SW=SW-TC*TX+TD*TY:SZ=SZ-TD*TX-TC*TY:NEXT J

```

```

1692 SW=SW-TC*TX:SZ=SZ-TC*TY:NEXT J
1700 WW(I)=SW:ZZ(I)=SZ:NEXT I
1710 LPRINT:LPRINT "WEIGHT VECTOR"
1720 FOR I=1 TO NCL
1722 LPRINT WW(I),ZZ(I):NEXT I
1724 LPRINT
1790 'LPRINT:LPRINT "INDICES FOR PICK-UP OF (R,S) AT EACH TIME POINT"
1799 '-----CALCULATE CANCELLED SIGNAL VS. TIME
1800 FOR NT=1 TO NTS
1810 U=U(NT):V=V(NT)
1820 FOR J=1 TO NCL'----ADD IN ALL TAP OUTPUTS
1822 NTJ=LTS+(J-1)*LTD
1830 N4=(NT-NTJ)*NSK+JCP
1840 IF N4<1 GOTO 1870
1842 IF N4>51 GOTO 1870
1850 'U7=R(N4):V7=S(N4):W7=WW(J):Z7=ZZ(J)
1852 U7=R(N4):W7=WW(J):Z7=ZZ(J)
1860 'U=U+FACT*(U7*W7-V7*Z7):V=V+FACT*(V7*W7+U7*Z7)
1862 U=U+FACT*U7*W7:V=V+FACT*U7*Z7
1870 'LPRINT N4;'-----A CHECK ON PROPER PICK-UP FROM WAVEFORM
1872 NEXT J:'LPRINT 'ACTIVATE HERE,1870,1790 (USES PAPER, TIME)
1880 U(NT)=U:V(NT)=V:NEXT NT
1890 GOSUB 1350'-----TO PLOT ON PRINTER
1892 'GOSUB 1450'-----OR ON PLOTTER
1898 GOTO 1100 'PRINT REF DATA AND STOP
2000 U=0:V=0'-----THIS SUB CALCULATES THE RAW SIGNAL
2020 FOR I=1 TO NSC'-----EACH SCATT
2030 Z9=Z[I]
2100 FOR J=1 TO NLS'-----EACH XMTR EL.
2210 D=D[I,J]+E[I]
2220 IF KEY=1 THEN 2300'---ELSE JUST GET MIN DELAY D5
2230 IF D>=D5 THEN 2250
2240 D5=D
2250 GOTO 2580
2300 N5=NSK*(INT((D5-D)/T1)+NT)'-----ID POINT ON WAVEFORM
2302 IF N5<1 THEN 2580
2306 IF N5>51 THEN 2580
2320 U5=R(N5):V5=S(N5)
2350 F=F[I,J]+G[I]'-----PHASE "ADVANCE" (NEG)
2360 C5=COS(F):S5=SIN(F)
2400 'U=Z9*(U5*C5-V5*S5)+U:V=Z9*(V5*C5+U5*S5)+V
2402 U=U+Z9*U5*C5:V=V+Z9*U5*S5
2580 NEXT J
2590 NEXT I
2592 IF ABS(U)<.000001 THEN U=.000001
2594 IF ABS(V)<.000001 THEN V=.000001
2599 RETURN
2600 J=INT(VNLS/2)'-----SCATTER TIMES (APPROX--REL. TO CTR. EL)
2602 IF J=0 THEN J=1
2604 DD=(KP-1)*T0
2610 FOR I=1 TO NSC
2630 D7=D[I,J]+E[I]-D5
2640 LPRINT D7,D7+DD
2670 NEXT I
2675 RETURN

```

```

2799 '-----PLOT A POINT ON PRINTER
2800 IF WX/AMAX>.017783 AND WX/AMAX<1 THEN 2830
2810 IF WX/AMAX<=.017783 THEN PLT$(3)=LB$:PLT$(4)=LB$:GOTO 2870
2820 IF WX/AMAX>1 THEN PLT$(76)=LB$:PLT$(77)=LB$:GOTO 2870
2822 IF WX=AMAX THEN NPLT=75:GOTO 2850
2830 DB=8.68589*LOG(WX/AMAX)
2840 NPLT=INT(DB*2+.5)+75
2842 IF NPLT>80 THEN NPLT=80
2844 IF NPLT<1 THEN NPLT=1
2850 TEMP$=PLT$(NPLT)
2860 PLT$(NPLT)=AST$
2870 FOR I=1 TO 80:LPRINT PLT$(I);:NEXT I:LPRINT
2880 PLT$(NPLT)=TEMP$
2900 PLT$(3)=BLK$:PLT$(4)=BLK$:PLT$(76)=BLK$:PLT$(77)=BLK$
2910 RETURN
3000 FOR I=1 TO NCL'-----CALC THE MATRIX TO BE INVERTED
3010 FOR J=1 TO NCL
3020 CAA=0:CBB=0
3100 FOR K=1 TO MP
3110 TPT=PP(K,I):TP=PP(K,J):'TQT=QQ(K,I):TQ=QQ(K,J)
3120 'CAA=CAA+TPT*TP+TQT*TQ:CBB=CBB+TPT*TQ-TQT*TP:NEXT K
3122 CAA=CAA+TPT*TP:NEXT K
3130 'AA#(I,J)=CAA:BB#(I,J)=CBB:NEXT J:NEXT I
3132 AA#(I,J)=CAA:NEXT J:NEXT I
3138 LPRINT:LPRINT "U(*,T) TIMES U":LPRINT'----PRINT IT
3140 FOR J=1 TO NCL
3150 FOR K=1 TO NCL
3160 WRAA=AA#(J,K):LPRINT USING "###.###";WRAA;:NEXT K:LPRINT:NEXT
J:LPRINT
3169 '-----SKIP THIS FOR REAL WAVEFORMS
3170 'FOR J=1 TO NCL
3180 'FOR K=1 TO NCL
3190 'WRBB=BB#(J,K):LPRINT USING "###.###^";WRBB;:NEXT K:LPRINT:NEXT J
3200 LPRINT:GOTO 1594
3400 FOR J=1 TO NCL'-----REPLACES SUB 6000 FOR REAL WAVEFORMS
3410 FOR K=1 TO NCL
3420 W#(NCL*(J-1)+K)=AA#(J,K):NEXT K:NEXT J
3430 GOSUB 6400
3440 FOR J=1 TO NCL
3450 FOR K=1 TO NCL
3460 CC#(J,K)=W#(NCL*(J-1)+K):NEXT K:NEXT J
3470 GOTO 1600
3899 '-----SEPARATE STORE OF SIGNAL FOR CANCEL (LEAVE ORIG.
ALONE)
3900 FOR NT=1 TO NTS
3910 U(NT)=B(NT):V(NT)=C(NT):NEXT NT:RETURN
3999 '-----DO PRODUCT OF U(*,T) AND TARGET VECTOR
4000 FOR J=1 TO NCL
4010 CXX=0:CYY=0
4100 FOR I=1 TO MP
4110 TPT=PP(I,J):TF=FF(I):'TQT=QQ(I,J):TG=GG(I)
4112 TG=GG(I)
4120 'CXX=CXX+TPT*TF+TQT*TG:CYY=CYY+TPT*TG-TQT*TF:NEXT I
4122 CXX=CXX+TPT*TF:CYY=CYY+TPT*TG:NEXT I
4130 XX(J)=CXX:YY(J)=CYY:NEXT J

```

```

4140 LPRINT:LPRINT "U(*,T) TIMES TARGET":LPRINT
4150 FOR I=1 TO NCL
4160 LPRINT XX(I);YY(I):NEXT I:LPRINT:GOTO 1660
4199 '-----
4200 LPRINT:LPRINT "DELAY LINE OUTPUT MATRIX":LPRINT
4210 FOR I=1 TO MP
4220 FOR J=1 TO NCL:LPRINT PP(I,J);:NEXT J:LPRINT:NEXT I:LPRINT
4230 'FOR I=1 TO MP
4240 'FOR J=1 TO NCL:LPRINT QQ(I,J);:NEXT J:LPRINT:NEXT I
4250 LPRINT:GOTO 1592
6000 'SUB ----- RETURN CC#+jDD#=INV(AA#+jBB#)
6002 FOR J=1 TO NCL '-----STORE AA# AS VECTOR W# FOR INVERSION SUB
6010 FOR K=1 TO NCL:W#(NCL*(J-1)+K)=AA#(J,K):NEXT K:NEXT J
6020 GOSUB 6400 '-----INVERT REAL MATRIX (STORED AS VECTOR)
6022 IF IDET=0 THEN STOP
6030 FOR J=1 TO NCL '-----RE-STORE INVERSE AS UU#
6040 FOR K=1 TO NCL:UU#(J,K)=W#(NCL*(J-1)+K):NEXT K:NEXT J
6050 FOR I=1 TO NCL '-----CC#=UU#*BB#
6060 FOR J=1 TO NCL:HH#=0#
6070 FOR K=1 TO NCL:HH#=HH#+UU#(I,K)*BB#(K,J):NEXT K:CC#(I,J)=HH#
6072 NEXT J:NEXT I
6080 GOSUB 6300 '-----DD#=BB#*CC# (FIRST TIME)
6090 FOR J=1 TO NCL '-----DD#=AA#+DD#
6100 FOR K=1 TO NCL:DD#(J,K)=AA#(J,K)+DD#(J,K):NEXT K:NEXT J
6110 FOR J=1 TO NCL '-----STORE DD# AS VECTOR W# FOR INVERSION
6120 FOR K=1 TO NCL:W#(NCL*(J-1)+K)=DD#(J,K):NEXT K:NEXT J
6130 GOSUB 6400 '-----INVERT IT
6132 IF IDET=0 THEN STOP
6140 FOR J=1 TO NCL '-----STORE INVERSE AS CC#
6150 FOR K=1 TO NCL:CC#(J,K)=W#(NCL*(J-1)+K):NEXT K:NEXT J
6160 GOSUB 6300 '-----DD#=BB#*CC# (SECOND TIME)
6170 FOR I=1 TO NCL '-----VV#=UU#*DD#
6180 FOR J=1 TO NCL:HH#=0#
6190 FOR K=1 TO NCL:HH#=HH#+UU#(I,K)*DD#(K,J):NEXT K:VV#(I,J)=HH#
6192 NEXT J:NEXT I
6200 FOR J=1 TO NCL '-----DD#=-VV#
6210 FOR K=1 TO NCL:DD#(J,K)=-VV#(J,K):NEXT K:NEXT J:RETURN
6300 FOR I=1 TO NCL '-----DD#=BB#*CC#
6310 FOR J=1 TO NCL:HH#=0#
6320 FOR K=1 TO NCL:HH#=HH#+BB#(I,K)*CC#(K,J):NEXT K:DD#(I,J)=HH#
6330 NEXT J:NEXT I:RETURN
6400 'INVERSION OF REAL MATRIX -- INPUT AND RETURNED AS VECTOR W#
6402 DET#=1:IDET=1:NK=-NCL '-----SEARCH FOR LARGEST ELEMENT
6410 FOR K = 1 TO NCL:NK=NK+NCL:LL(K)=K:MM(K)=K:KK=NK+K:BW#=W#(KK)
6420 FOR J = K TO NCL:IZ=NCL*(J-1)
6430 FOR I = K TO NCL:IJ=IZ+I:IF ABS(BW#) >= ABS(W#(IJ)) GOTO 6450
6440 BW#=W#(IJ):LL(K)=I:MM(K)=J
6450 NEXT I:NEXT J
6460 J=LL(K):IF J <= K GOTO 6490 '-----INTERCHANGE ROWS
6470 KI=K-NCL
6480 FOR I=1 TO
NCL:KI=KI+NCL:HH#=-W#(KI):JI=KI-K+J:W#(KI)=W#(JI):W#(JI)=HH#
6482 NEXT I
6490 I=MM(K):IF I <= K GOTO 6530 '-----INTERCHANGE COLUMNS
6500 JP=NCL*(I-1)

```

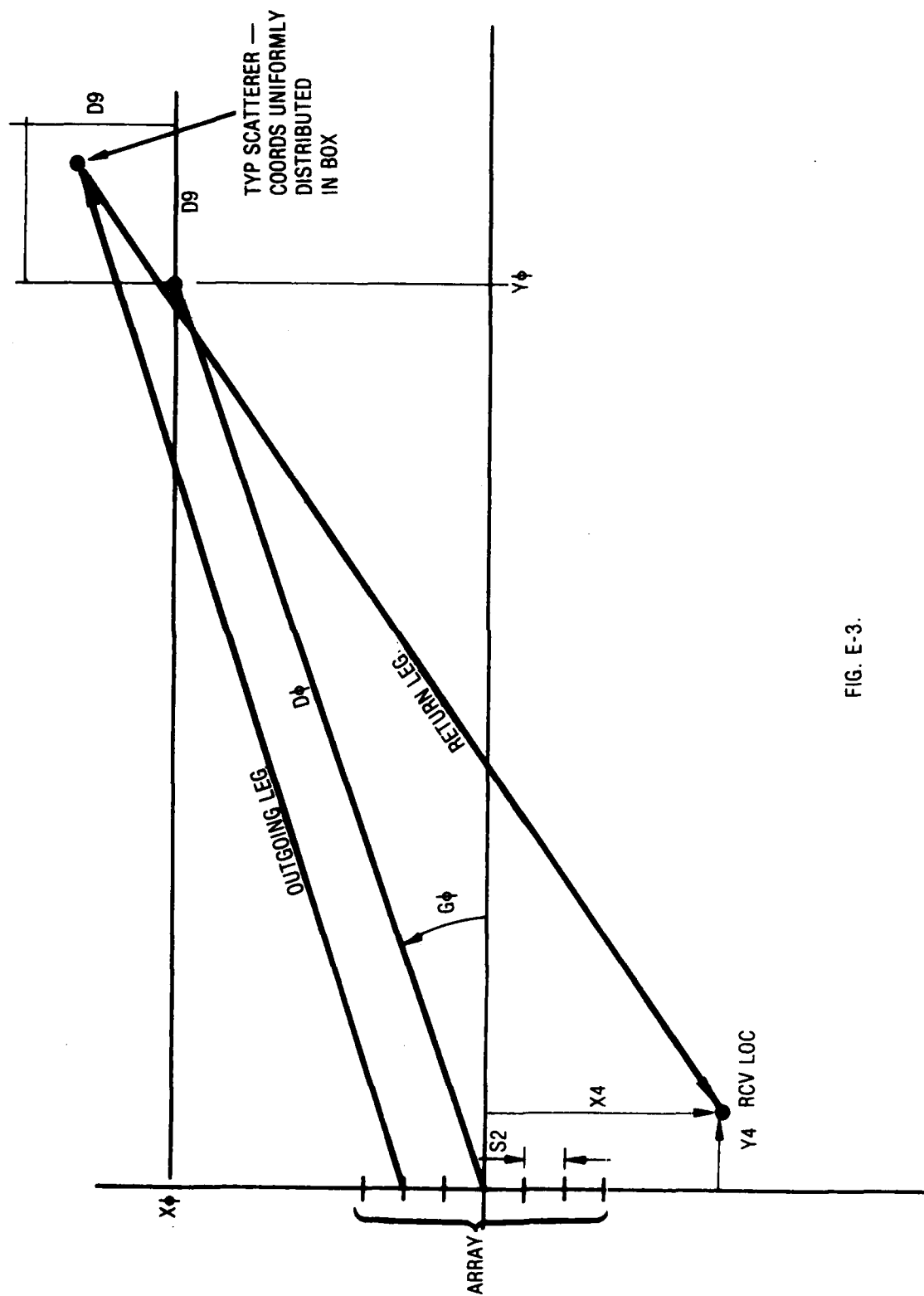
```

6510 FOR J=1 TO
NCL:JK=NK+J:JI=JP+J:HH#=-W#(JK):W#(JK)=W#(JI):W#(JI)=HH#:NEXT J
6520 '-----DIVIDE COLUMN BY MINUS PIVOT (VALUE IN BW#)
6530 IF BW# = 0 THEN IDET=0:LPRINT "DETERMINANT IS ZERO":RETURN
6540 FOR I = 1 TO NCL:IF I = K GOTO 6560
6550 IK=NK+I:W#(IK)=W#(IK)/(-BW#)
6560 NEXT I
6570 FOR I = 1 TO NCL:IK=NK+I:HH#=W#(IK):IJ=I-NCL '-----REDUCE
MATRIX
6580 FOR J=1 TO NCL:IJ=IJ+NCL:IF I = K OR J = K GOTO 6600
6590 KJ=IJ-I+K:W#(IJ)=HH#*W#(KJ)+W#(IJ)
6600 NEXT J:NEXT I:KJ=K-NCL '-----DIVIDE ROW BY PIVOT
6610 FOR J=1 TO NCL:KJ=KJ+NCL:IF J = K GOTO 6630
6620 W#(KJ)=W#(KJ)/BW#
6630 NEXT J:DET#=DET#*BW#:W#(KK)=1/BW#:NEXT K:K=NCL
6640 K=K-1:IF K <= 0 THEN LPRINT "DETERMINANT IS ";DET#:RETURN 'FINAL
EXCH.
6650 I=LL(K):IF I <= K GOTO 6680
6660 JQ=NCL*(K-1):JR=NCL*(I-1)
6670 FOR J=1 TO
NCL:JK=JQ+J:HH#=W#(JK):JI=JR+J:W#(JK)=-W#(JI):W#(JI)=HH#:NEXT J
6680 J=MM(K):IF J <= K GOTO 6640
6690 KI=K-NCL
6700 FOR I =1 TO
NCL:KI=KI+NCL:HH#=W#(KI):JI=KI-K+J:W#(KI)=-W#(JI):W#(JI)=HH#
6710 NEXT I:GOTO 6640
9999 END'+++++++THIS IS THE END--"9999 END'+++..."ETC

```

At line 200, following preliminaries, the reference waveform [R+jS] is read from the data table and stored together with setting of the delta t's for the reference waveform and the signal waveforms, their ratio NSK, and the designated key point (appropriate peak) of the waveform. For ID and labeling, the bandwidth and pulse width are also stored. NSK permits different granularity of the reference and signal waveforms. Waveforms generated by program FILT are available on disk as files identical in form to lines 200-280. The editor permits a block substitution of any selected waveform file before loading and running. This waveform, with specified bandwidth and driving pulse width, is the source for the reflected signal components and the delay line outputs.

At 300, the RN generator is initialized and can be "slipped" to produce different random samples of scatterer arrays. At 310 the scatterer and array parameters are entered. Run time dimensioning is invoked at 350. Beginning at 400, the geometry of the group of scatterers is calculated and converted to time delays and phase delays (principal value) and stored. Figure E-3 shows the geometry of the scatterers and antennas. Each combination of array element and scatterer determines a separate path. At 800, subroutine 2000 is called (with KEY=0) to get the delay over the shortest path. This is used as the time reference since no reflection can precede this instant. These stored data provide the basis for signal calculation.



Starting at 1000, the "raw signal" or total return via all paths is computed by calling subroutine 2000 with KEY=1 for each time point, at intervals  $\Delta t = T1$ , and stored as vector  $[B+jC]$ . For point 1, a "low" but non-zero value is assigned to prevent trouble with the log function. This point is the instant at which the first point of the waveform (with zero amplitude in all cases) arrives at the receiving point via the shortest path. The maximum value of amplitude WX is saved as AMAX to be used as the reference level.

Because of its central role, subroutine 2000 is discussed next, out of order. This subroutine defines each two-way path via a double loop, "each scatterer" and "each array element", and the total path delay is found. As stated above, if KEY=0 (call at 800) the minimum, D5 is merely updated. If KEY=1 (call at 1040), the point of the reference waveform which is just returning at time point NT via this path is identified (N5, lines 2300-2306) and its value,  $R(N5) + jS(N5)$ , or  $R(N5)$  only in MST4RL, is read from the stored vector  $[R + jS]$ . This is necessarily an approximation in view of the granularity of the data. There appears to be the assumption here of zero (or  $2k$ ) phase shift over the delay path. At a single frequency this is justified simply in that a different phase shift would merely change the required weight (i.e., its angle). This remains true for the carrier factor of a broad-band signal. The phase shift at side-band frequencies cannot then be zero since there must be a phase slope in view of the delay. However, if the delay path is non-dispersive, so that the complex envelope is merely delayed but otherwise preserved, the phase shifts at side-band frequencies are implicit in this distortionless behavior. Put another way, the expression  $A(t) \cos W.t$ , where  $A(t)$  is broad-band but real, gives the correct instantaneous result,

notwithstanding a phase shift dependent on frequency over the band occupied by the signal, provided that shift is linear with frequency (constant delay). The principal values of phase shift previously stored for each of the two path legs,  $F(I,J)$  and  $G(I)$ , are used to find the phase of the return (relative to zero at carrier frequency) and applied to the waveform value to get the contribution of the two-way path. These contributions are summed over all paths to get the total raw signal at the given time point and returned by the subroutine as the phasor  $U + jV$ . A lower limit is placed on  $U$  and  $V$  as in the case of point 1.

Returning now to proper sequence in the main course of the model, at 1200 a working copy of the raw signal vector is made,  $U + jV$  (subscripted vector) via sub 3900 and plotted via sub 1350 using the printer, or subs 1400 and 1480 using the plotter.

The cancellation process at 1500 begins with input of the selected time points to be "cancelled", or minimized in the LMS sense; i.e., the start, end, step, and number (MP), not all of which are independent of course. Similarly the number of taps (NCL) is specified. These inputs are followed by more run time dimensioning. At 1550, the  $MP \times NCL$  delay line output matrix  $[PP + jQO]$ , denoted analytically as " $u$ ", is constructed. The same argument as to phase shifts applies here as in the case of path delays. Therefore, the reference waveform is merely picked up using the correct index  $I7$  (lines 1560-1564) depending on tap and cancel point. In MST4RL this matrix remains real ( $QO$  identically zero). A patch to 4200 prints this matrix and another patch to

3000 performs the multiplication  $N^*T_u$ , where " $*T$ " means conjugate complex transpose, and also prints this product  $[M^*+jBB^*]$ , of size  $NCL \times NCL$ . A call to 6000, or a patch to 3400 in the case of MST4RL where this matrix is real, inverts it to give  $[u^*T_u]^{-1}$ , or  $[CC^* + jDD^*]$ , also of size  $NCL \times NCL$ , completing a major part of the weight calculation.

At 1600 the vector  $[FF+jGG]$  of length MP consisting of the raw signal at the cancel points is constructed. This has been denoted the "target vector" because the object is to "shoot it down" to a low level. A patch to 4000 finds the product of  $u^*T$  (size  $NCL \times MP$ ) and the target vector. This product  $[xx+jYY]$  is another vector, of length NCL.

At 1660 the final product including the negative sign produces the weight vector  $[WW+jZZ]$  of length NCL, given by  $-[U^*T_x U]^{-1} \times [U^*T_x(\text{target vector})]$ , and this is printed. The cancelled or minimized signal can now be computed.

At 1800, the raw signal is summed with all the weighted tap outputs in a double loop: "each time" and "each tap". The output from each tap at each time is identified by index N4 (lines 1830-1842) and multiplied by the weight at that tap to get the increment of the signal at that time. The computation is finished at 1872. (The print-out of index values N4 as noted in 1872 could be activated if desired but this has been used mainly in debugging and verification.) A call to 1350 plots the curve on the printer; alternatively, calling 1480 plots on the plotter. Finally, a patch to 1100 prints a summary of data, including primarily the mean power level and the worst point level of the minimized residual signal relative to the original peak voltage AMAX, obtained by a scan over the time points. The program stops here.

Several subroutines and patches used in support of the "main line" are self-explanatory. As used here "patch" means a jump out, usually ending with a jump back, but not a GOSUB...RETURN. Several "subs" are called only once and need not have been called by GOSUB. Of the subs, a key one at 6400 inverts a real matrix (stored first as a vector) using double precision. This well-known routine is not further discussed.

Sub 6000 inverts a complex matrix. For brevity, abbreviate "AA\*" as A, etc. What is required is  $[A+jB]^{-1}$ . This can be expanded to the expression

$$[A+BA^{-1}B]^{-1} + j[-A^{-1}B(A+BA^{-1}B)^{-1}]$$

In this discussion, since all entities are matrices, the rigid convention of enclosing a matrix in [] is relaxed,

To get this as  $C+jD$ , the following procedure is used, all named matrices being real, of the same size, and square:

$$\begin{array}{lll} U & = & A^{-1} \\ C & = & UB \\ D & = & BC \\ D & = & A+D \\ C & = & D^{-1} \\ D & = & BC \\ V & = & UD \\ D & = & -V \end{array}$$

Matrices C and D are used for intermediate storage as well as for the final inverse. Additional temporary storage, U and V, is also required. The subroutine carries out this sequence, calling sub 6400 twice. The product  $D=BC$  also appears twice and is conveniently put as a sub at 6300. This and other matrix multiplications at 3000 and 4000 are applications of standard definitions.

The program provides two plotting facilities. That which is active as shown in the listing (at 1350) uses the printer. Each time point results in a print line representing amplitude on a scale of -35 to zero dB at 1/2 dB per character. The line content is initialized (line 172) to blanks except a period every 10 characters (5 dB). Subroutine 2800 converts the amplitude WX to dB relative to the reference level AMAX, rounded to 1/2 dB, and substitutes "\*" for the initialized blank or period at the proper position. If the value is out of range, "\*\*\*" is substituted for two blanks just beyond the appropriate scale limits, -35 or zero. The alternative plot mode employs a plotter controller which responds to prescribed control characters. This controller requires that a driver be inserted into memory. This is done by the main program at (52-86), using the POKE statement. Commands are then issued via the LPRINT statement (same as the printer). A specified location (65052 as shown) is set to steer commands to the printer or reset to command the plotter controller. The plotter is initially off (line 88) and turned on only when needed, then turned off again, in subs 1400 and 1480. At 1400, the plot scales are established and axes drawn. At 1480, the dB values are computed and plotted. Functions FNX and FNY employ the scale values to compute the proper coordinate values for the controller.

### 3. Model of Gradient Solution

Like all the models, the gradient model MSTGR differs essentially from MST4 only in the cancellation process (coded in the region 1500 up to less than 1900 in all models), plus a minor difference in the reference data print-out area (from 1100 to less than 1200 in all models). It happens that the real only simplification analogous to that of MST4RL has not been made in MSTGR. As will be seen this would seem to offer very little in reduced running time and nothing in storage requirements. Listing E-2 combines the two segments which differ from those of MST4. MSTGR does not include coding that appears in MST4 in lines 3000 to the end, all of which except 3900-3910 are involved in the matrix solution but not the gradient approach. The test of sub 3900 is done a little differently in MSTGR. One other small difference is that storage is so minimal that run-time dimensioning of the weight vector is not used; dimensions are set in line 140 to allow a tap at each of the time points (weight vector [WW+jZZ]).

The gradient solution consists of the iterative updating of the weight vector according to the "steepest descent" algorithm, starting with an arbitrary weight taken in that model as zero. The weight updating alternates with a recalculation of the residual signal using the new weights against the raw signal. The weight updating process for each tap consists in "adding"  $\Delta$  (but with a negative "gain factor") the integral of the product of the latest improved version of the residual signal and the (unweighted) delay line tap

## LISTING-2

## REFERENCE DATA PRINT-OUT -- GRADIENT/LMS SOLUTION

```

1100 LPRINT "MEAN POWER REL. TO ORIG. MAX (DB) " ; DBMN
1110 LPRINT "WORST POINT REL. TO ORIG. MAX (DB) " ; DBWRST ; "AT PT.
";NRST
1120 RETURN
1150 LPRINT:LPRINT "BW,PW,T0,T1,NSK,JCP,RN SLIP
";BW;PW;T0;T1;NSK;JCP;JRN
1152 LPRINT "SCATTERERS " ;NSC;D0;G0;D9
1154 LPRINT "ANTENNAS " ;F7;S7;NLS;X4
1156 LPRINT "TAPS:START,NR.,STEP; GAIN FACTOR";LTS;NCL;LTD;FACT
1158 LPRINT "TAP SPACING ";LTD*T1;" REF LEVEL 'VOLTS' ";AMAX
1160 RETURN

```

## CANCELLATION PROCEDURE -- GRADIENT/LMS SOLUTION

```

1500 INPUT "TAPS:START,END,STEP,NR; GAIN FACTOR";LTS,LTE,LTD,NCL,FACT
1530 IFRST=1:KFRST=1
1550 FOR J=1 TO NCL
1560 WW(J)=0:ZZ(J)=0:NEXT J
1580 ISKP=1
1590 LPRINT:LPRINT "GAIN FACTOR IS ";FACT:LPRINT
1600 SUM=0:VWRST=0:NRST=999
1610 FOR NT=1 TO NTS
1620 UT=B(NT):VT=C(NT)
1622 IF KFRST=1 GOTO 1670
1630 FOR J=1 TO NCL
1640 I7=NSK*(NT-LTD*(J-1)-LTS)+JCP
1642 IF I7<1 GOTO 1662
1644 IF I7>51 GOTO 1662
1650 UI=R(I7):VI=S(I7):UW=WW(J):VW=ZZ(J)
1660 UT=UT+UI*UW-VI*VW:VT=VT+VI*UW+UI*VW
1662 NEXT J
1670 U(NT)=UT:V(NT)=VT
1672 WX2=UT*UT+VT*VT:WX=SQR(WX2)
1680 SUM=SUM+WX2:IF VWRST<WX THEN VWRST=WX:NRST=NT
1690 NEXT NT
1692 KFRST=0
1700 DBMN=8.68589*LOG(SQR(SUM/NTS)/AMAX)
1702 DBWRST=8.68589*LOG(VWRST/AMAX)
1704 GOSUB 1100
1706 IF ISKP=1 GOTO 1750
1710 INPUT "PLOT";PLOT$:IF PLOT$<>"Y" GOTO 1730
1720 GOSUB 1350
1722 'IF IFRST=1 THEN IFRST=0:GOSUB 1400
1724 'GOSUB 1480
1730 INPUT "CONTINUE";QCON$:IF QCON$="Y" GOTO 1750
1740 GOSUB 1150:STOP
1750 FOR J=1 TO NCL
1760 UR=0:VR=0
1770 FOR NT=1 TO NTS
1780 I7=NSK*(NT-LTD*(J-1)-LTS)+JCP
1782 IF I7<1 GOTO 1810
1784 IF I7>51 GOTO 1810
1790 UI=R(I7):VI=S(I7):UT=U(NT):VT=V(NT)
1800 UR=UR+UI*UT-VI*VT:VR=VR+VI*UT+UI*VT
1810 NEXT NT
1820 WW(J)=WW(J)-FACT*UR:ZZ(J)=ZZ(J)-FACT*VR
1822 IF ISKP=1 GOTO 1840
1830 LPRINT WW(J);ZZ(J)
1840 NEXT J:LPRINT:GOTO 1600

```

output. This integral (or finite summation as implemented) is a measure of the correlation between the current residual signal and the tap output. A high (positive) correlation indicates, for example, that a correspondingly large negative change in the weight at that tap will be helpful for the next try.

The iterative nature of the cancellation process in MSTGR is illustrated in Figure E-4, a kind of flow diagram. Once the circle is entered at line 1600 the process repeats indefinitely until interrupted. At 1500 the set of taps is defined, the weight vector is initialized to zero and some controls are set. (IFRST is used only for the plotter, to prevent repetition of scaling and axis plotting.) At 1600, the entry to the iterative cycle, a loop, "each time", initializes a working phasor  $U + jV$  to the raw signal value at that time. If  $KFRST=1$ , this is merely stored as the "cancelled signal" in vector  $U + jV$  (the job done in MST4 by sub 3900) thus making available a plot of the raw signal, if desired. If  $KFRST$  is zero, the raw signal value is subjected to the cancellation process using the current weight vector before storing as the cancelled signal for the given time point. In either case, the signal vector is scanned over the time loop for mean and worst point and when the loop closes  $KFRST$  is set to zero. At 1700, the mean and worst point are converted to dB and printed by sub 1100. This provides monitoring of the convergence process.

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ADAPTIVE CANCELLATION TECHNIQUES(U) RAYTHEON CO GOLETA  
CA ELECTROMAGNETIC SYSTEMS DIV 21 NOV 83  
N00014-81-C-2575

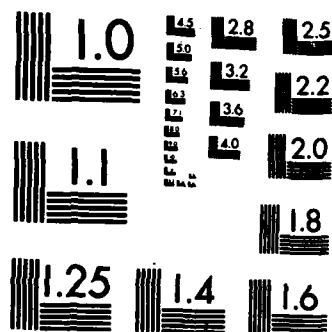
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END



MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

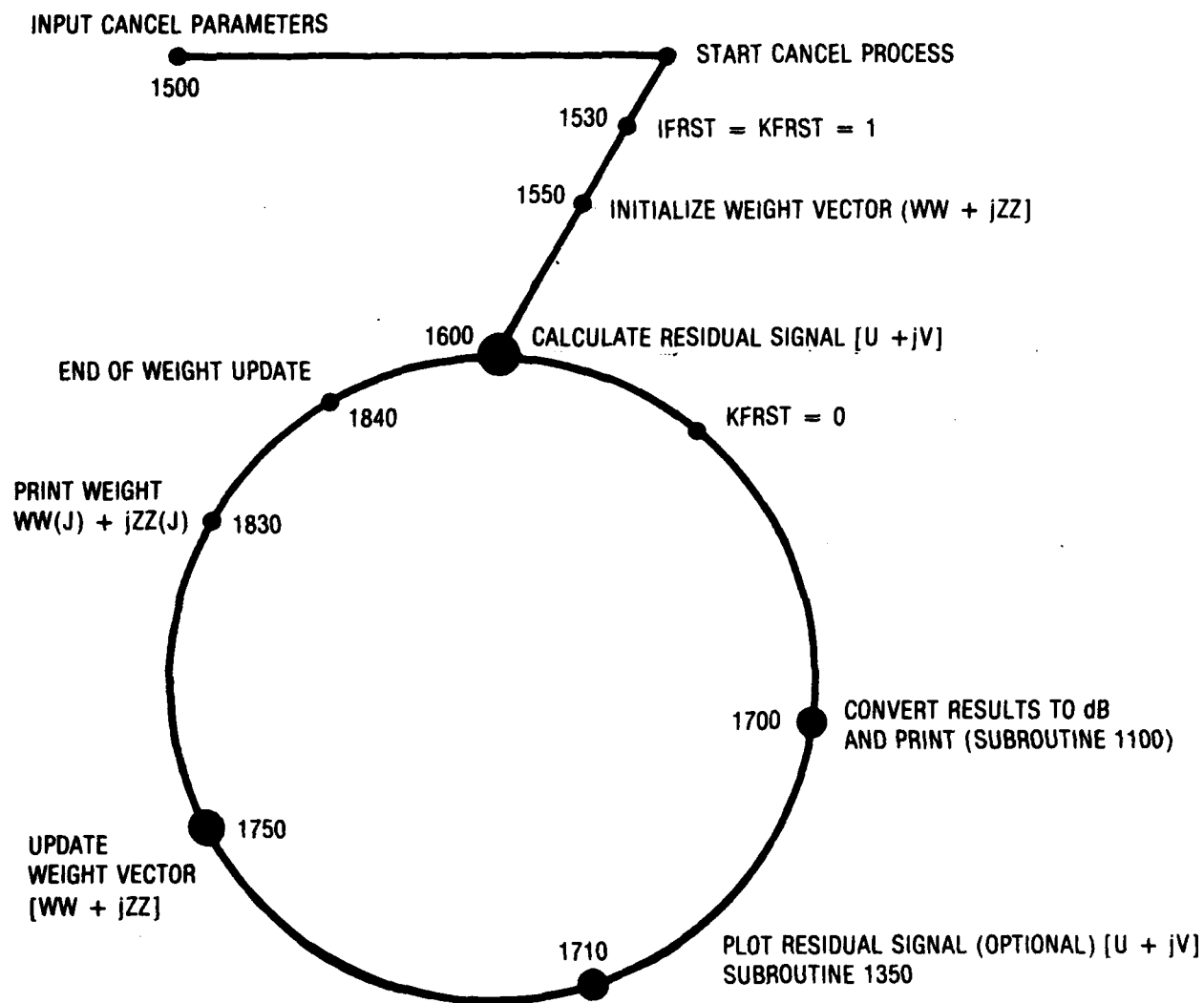


FIGURE E-4

ISKP is in effect a "sense switch". At any time the program can be interrupted and ISKP can be changed to zero to suspend skipping. This enables a plot option at 1710, a continue option at 1730, and the weight print-out at 1830, as each is encountered. After setting or resetting ISKP, the program can be reentered with a CONT command. The purpose of ISKP is mainly to avoid the undesired use of time and paper by restricting the printing to the mean and worst point data. These two data are the basis for deciding on changes in the gain factor, which controls the iterative loop behavior. The best time to interrupt is at completion of weight updating, indicated by printing of the last weight or by printer line feed if ISKP=1. At this time recalculation of the residual signal vector  $[U+jV]$  is in progress. Having elected not to continue, the program stops when it reaches line 1740 after printing reference data (sub 1150). It is then possible to change the "gain factor" manually ("FACT=...") and GOTO 1530, thus starting a fresh attempt at cancellation, or to GOTO 1500 allowing a change of taps as well. The interrupt and CONT or GOTO nn capabilities combined with manual entry of ISKP and other variables give flexibility to experiment with the control loop while retaining the raw signal vector  $[B+jC]$  unchanged.

At line 1750 the weight vector  $[WW+jZZ]$  is updated by performing the required integration at 1770 and the correction at 1820. The negative gain factor is seen explicitly in line 1820; i.e., FACT is entered as intrinsically positive (line 1500). Depending on ISKP, the weight vector is printed, thus saving some time and paper if not needed.

The loop then recycles to line 1600. That completes the description of MSTGR. As to the real-only simplification previously described, if this were done in MSTGR it would show up only in lines 1650-1660 and in lines 1790 and 1800. In each case VI would be zero, deleting two terms each in lines 1660 and 1800. It is doubtful that running time would be noticeably shortened. However, there is no obvious argument against this change when time permits.

#### 4. SEQUENTIAL SWEEP MODEL

The sweep method is a conceptually simple attack on the cancellation problem. It is an attempt to make something of the clearly successful but totally inadequate single-point cancellation, following the philosophy that "if it works once, try it again". Unfortunately, as the sweep proceeds, each new single-point cancellation tends to undo what has been accomplished up to that point. Figure E-5 shows how the interference between taps with overlapping waveform outputs takes place. Assume that the taps preceeding tap j have been somehow adjusted, leaving a certain residual signal. Taps j and higher are not yet "weighted in" to the summation. If the tap j weight is now adjusted so that its key point cancels the residual wave at the time this key point is emerging from the delay line, the earlier non-zero parts of the tap j wave will affect whatever results were previously obtained via earlier taps, which were set in absence of the tap j wave. In addition, later parts of the tap j wave will now be changing, and in general augmenting, the later parts of the residual signal making it more difficult to cancel. If there were no "earlier parts" of the wave, the retroactive effect would not occur. Thus one would try to use a leading edge "shoulder point" as the key point. By consistently applying

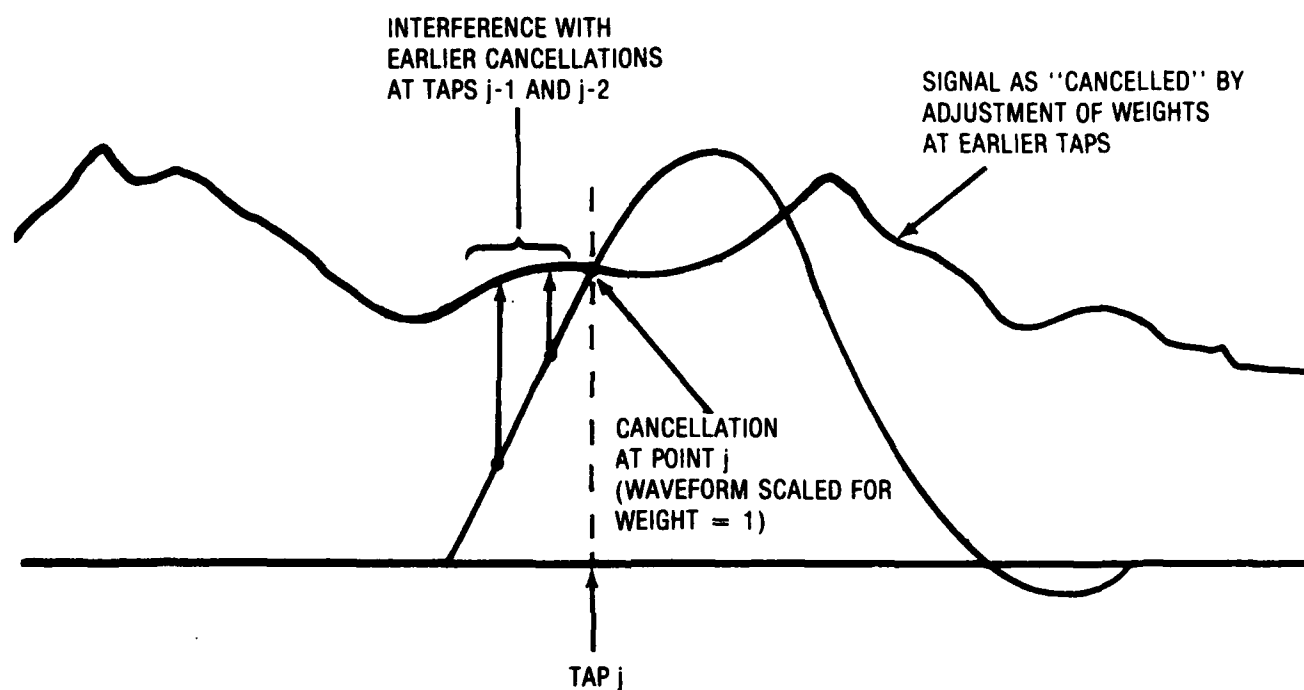


FIG. E-5.

less than 100 percent of the weighted tap output needed to cancel, the disruption of earlier cancellations might be mitigated, though all cancellations are then imperfect. A possible improvement might result from subdividing the taps into interlaced sets and sweeping through these sets in sequence, a simple repetition of the sweep might also be used.

Listing E-3 shows the cancellation part of the sweep model MST1. It is best to look first at the subroutine at 1600. Given the current partially cancelled signal vector  $[U+jV]$  (time function of length NTS), this sub carries out the single-point cancellation at the time point N7. This implies the existence of a tap from which key point JP of the waveform is emerging at this time. The sub finds the weight, which when multiplied by the key point value of the waveform yields a value which cancels the signal at that time; then it scans the whole signal wave vs. time point-by-point, adding the product of this fixed weight (with percentage FACT) times whatever part of the waveform is then emerging from the tap, to obtain the new version of the signal vector.

This subroutine is called in a double loop, "each sweep" and "each step", specified by start, end, step, and number of sweeps. On the first (or any odd) sweep, the time point N7 used in calling the sub merely steps from start to end in the normal manner, since line 1524 is satisfied. On even sweeps, N7 is advanced one half step, thus implying the existence of a second, interleaved set of taps. Thus an even number of sweeps implies half the tap spacing. This interlace could be deleted by inactivating lines 1524, 1526. The point is largely moot however, since multiple sweeps are not of major interest.

LISTING E-3

CANCELLATION PROCEDURE IN MODEL MST1 -- SWEEP SOLUTION

```

1500 INPUT "SW(STRT,END,STEP),NR SWS";L4,L5,L6,NSW
1502 INPUT "SHLDR PT.,ADJ.FACT.";JP,FACT
1504 LPRINT L4;L5;L6;NSW;JP;FACT
1506 LPRINT:LPRINT "TIME AT CANCEL POINTS":LPRINT
1510 FOR L=1 TO NSW
1520 FOR K=L4 TO L5 STEP L6
1522 N7=K          'POINT TO CANCEL (TENT.)
1524 IF L MOD 2=1 THEN 1530 'ODD SWEEP
1526 N7=N7+INT(L6/2+.1) 'IF EVEN SWEEP
1530 GOSUB 1600
1532 LPRINT (N7-1)*T1
1540 NEXT K
1542 LPRINT
1550 NEXT L
1565 GOSUB 1350
1566 REM GOSUB 1450
1580 GOTO 1100
1600 '              SUB -- CANCEL AT THE POINT
1610 U=U[N7]:V=V[N7]
1640 RJP=R(JP):SJP=S(JP):DENOM=RJP*RJP+SJP*SJP
1660 U8=-(U*RJP+V*SJP)/DENOM:V8=-(V*RJP-U*SJP)/DENOM
1700 FOR N=1 TO NTS 'RECALC SIG AT EACH TIME
1710 N4=(N-N7)*NSK+JP 'CANCEL WAVE POINT AT THIS TIME
1720 IF N4<1 THEN 1820 'ZERO HERE
1740 IF N4>51 THEN 1820 'AND HERE
1760 RN4=R(N4):SN4=S(N4)
1780 U7=U8*RN4-V8*SN4:V7=V8*RN4+U8*SN4
1800 U[N]=U[N]+U7*FACT:V[N]=V[N]+V7*FACT
1820 NEXT N
1840 RETURN

```

## 5. SIMULTANEOUS EQUATION MODEL

This solution to the cancellation problem may be considered a special case of the matrix LMS solution in which the number of taps is the same as the number of cancel time points. Obviously, the exact cancellation satisfies the LMS criterion in this case. In procedural terms, this approach is simpler than the matrix/LMS approach because (1) the delay line output matrix is already square so that the multiplication written analytically as  $U^T \cdot U$  is not needed to get an invertible matrix of size  $n$  (for  $n$  taps) and (2) the "target vector" is already of length  $n$  so that the multiplication  $U^T \cdot (\text{target vector})$  is not needed to reduce the length from a larger value,  $m$ . It is merely necessary to invert the delay line matrix itself and multiply times the target vector (with negative sign) to get the weight vector.

Listing E-4 covers the cancellation procedure for the simultaneous equation solution. At 1500 one set of numbers defines both taps and cancel points and run-time dimensioning is invoked. At 1550 a double loop constructs the delay line output matrix  $[AA+jBB^*]$ , ready for inversion to yield  $[CC+jDD^*]$  in sub 6000 (which is identical to that subroutine in MST4. At 1600 the target vector  $[XX+jYY]$  is constructed. At 1670 the weight vector is found as the (negative) product of  $[CC+jDD^*]$  and  $[XX+jYY]$ , and printed at 1720. By-passing of unneeded, complex calculations in view of the real-only reference waveforms has not been installed in MST3. This would have less effect than in MST4RL because it would not affect anything but the construction and inversion of the delay line output matrix. At 1800, the calculation of the cancelled signal is substantially identical to that in MST4, with a slight simplification in ID of the reference waveform point emerging from a given tap at a given time.

LISTING E-4

CANCELLATION PROCEDURE IN MST3 -- SIMULTANEOUS EQUATION SOLUTION

```

1500 INPUT "TAPS:START,END,STEP,NR";LCS,LCE,LCD,NCL:NCL2=NCL*NCL
1510 DIM AA#(NCL,NCL),BB#(NCL,NCL),CC#(NCL,NCL),DD#(NCL,NCL)
1520 DIM UU#(NCL,NCL),VV#(NCL,NCL),XX(NCL),YY(NCL),W#(NCL2)
1530 DIM WW(NCL),ZZ(NCL),LL(NCL),MM(NCL)
1540 LPRINT:LPRINT "INDICES FOR PICK-UP OF R,S FOR MATRIX"
1550 FOR I=1 TO NCL
1552 FOR J=1 TO NCL
1560 I7=(I-J)*LCD*NSK+JCP
1562 IF I7<1 THEN U7=0:V7=0:GOTO 1580
1564 IF I7>51 THEN U7=0:V7=0:GOTO 1580
1570 U7=R(I7):V7=S(I7)
1580 AA#(I,J)=U7:BB#(I,J)=V7 'CANCEL MATRIX=AA#+jBB#
1590 LPRINT I7;:NEXT J:LPRINT:NEXT I
1600 LPRINT:LPRINT "RAW SIGNAL AT CANCEL POINTS" '(TARGET VECTOR)
1610 FOR I=1 TO NCL
1620 I7=LCS+(I-1)*LCD
1630 U7=U(I7):V7=V(I7)
1632 XX(I)=U7:YY(I)=V7 'XX+jYY IS "TARGET VECTOR"
1640 LPRINT I7;U7;V7:NEXT I:LPRINT
1650 GOSUB 6000 'GET CC=jDD=INV(CANCEL MATRIX)
1660 'GET WW+jZZ=WEIGHT VECTOR=-(INV(CANCEL MATRIX)*TARGET VECTOR)
1670 FOR I=1 TO NCL:SW=0:SZ=0
1680 FOR J=1 TO NCL:TC=CC#(I,J):TD=DD#(I,J):TX=XX(J):TY=YY(J)
1690 SW=SW-TC*TX+TD*TY:SZ=SZ-TD*TX-TC*TY:NEXT J
1700 WW(I)=SW:ZZ(I)=SZ:NEXT I
1710 LPRINT:LPRINT "WEIGHT VECTOR"
1720 FOR I=1 TO NCL
1722 LPRINT WW(I),ZZ(I):NEXT I
1724 LPRINT
1790 'LPRINT:LPRINT "INDICES FOR PICK-UP OF (R,S) AT EACH TIME POINT"
1800 FOR NT=1 TO NTS
1810 U=U(NT):V=V(NT)
1820 FOR I=1 TO NCL
1830 N4=(NT-LCS-(I-1)*LCD)*NSK+JCP
1840 IF N4<1 GOTO 1870
1842 IF N4>51 GOTO 1870
1850 U7=R(N4):V7=S(N4):W7=WW(I):Z7=ZZ(I)
1860 U=U+(U7*W7-V7*Z7):V=V+(V7*W7+U7*Z7)
1870 'LPRINT N4;
1872 NEXT I:'LPRINT 'ACTIVATE HERE,1870,1790 (USES PAPER, TIME)
1880 U(NT)=U:V(NT)=V:NEXT NT
1890 GOSUB 1350
1892 REM GOSUB 1450
1898 GOTO 1100 'PRINT REF DATA AND STOP

```

## 6. WAVEFORM TEST MODEL MSTX

An important issue is the possibility of "calibrating the cancellation process by use of a particular waveform and still achieving adequate cancellation when other jamming waveforms are actually radiated; e.g., a noise spot. MSTX does not perform a cancellation adjustment on its own but merely leads the weight vector from a data table transferred into it, which has been generated with a different waveform by one of the other models. This model generates the raw signal from its own waveform and computes the cancelled signal in the same way as the other models. A valid test requires MSTX to be run with the identical set of scatterers and antenna configuration and the same set of taps as the "calibrating" model. Listing E-5 shows the weight vector input portion of MSTX along with the necessary inputs for defining the taps, etc. The weight vector data from several specific runs of a calibrating model (specifically MST4 or MST4RL) have been stored on disk files which can be quickly substituted using the editor, before running MSTX.

## 7. FILTER RESPONSE PROGRAM

An analysis made on another program led to computation of responses of bandpass filters to prescribed input pulses, limited to AM only but permitting carrier frequencies offset relative to the band center. The filters are "poles-only" but the analysis could be extended to include zeroes. For present purposes the offset was set at zero. The program was originally written for the 370 computer. It has been adapted to the HP 9830 and also the S-100 computer used in the present modelling task. The program was intended for plotting and the 9830 version does so. On the S-100, print-out of the data has been

used for manual entry into brief disk files which can then be "edited into" the simulation models. Listing E-4 is the current version for the S-100. Appendix is the original analysis provided in explanation of the program. In the Introduction part of the Appendix \_\_\_\_, the "output envelope" should be interpreted as the complex envelope, not merely the amplitude. The "real-only" filters used in the current modelling do not exhibit phases other than zero and  $\pi$  when driven by center-frequency pulses, but the program handles the general case provided the input signal does not have FM.

LISTING E-5

CANCELLATION PROCEDURE IN MSTX -- USING EXTERNAL WEIGHT VECTOR

```

1500 LTS=4:LTE=58:LTD=6:NCL=10
1510 DIM WW(NCL),ZZ(NCL)
1600 FOR I=1 TO NCL
1610 READ WW(I),ZZ(I):NEXT I
1620 DATA 1.07262,-.13553
1621 DATA -1.87321,.82289
1622 DATA 8.19343,-1.7444
1623 DATA 5.85207,2.14795
1624 DATA -4.63885,-3.75577
1625 DATA -7.75202,-6.97995
1626 DATA -.04211,.30663
1627 DATA .1825,3.6685
1628 DATA 2.00235,10.9158
1629 DATA -2.41184,-.62558
1650 LPRINT "CAL CASE 5220-0925:T0=.4;JCP=16;TAPS 4,58,6
(10);250/7.11"
1660 PRINT "FOR CAL CASE T0 AND JCP ARE .4 16"
1700 PRINT "FOR WAVEFORM T0 AND JCP ARE ";T0;JCP
1710 INPUT "WHAT JDP";JDP
1720 LPRINT:LPRINT "JDP: ";JDP:LPRINT
1800 FOR NT=1 TO NTS
1810 U=U(NT):V=V(NT)
1820 FOR J=1 TO NCL
1822 NTJ=LTS+(J-1)*LTD
1830 N4=(NT-NTJ)*NSK+JDP
1840 IF N4<1 GOTO 1870
1842 IF N4>51 GOTO 1870
1850 U7=R(N4):V7=S(N4):W7=WW(J):Z7=ZZ(J)
1860 U=U+FACT*(U7*W7-V7*Z7):V=V+FACT*(V7*W7+U7*Z7)
1870 'LPRINT N4;
1872 NEXT J:'LPRINT 'ACTIVATE HERE,1870,1790 (USES PAPER, TIME)
1880 U(NT)=U:V(NT)=V:NEXT NT
1890 GOSUB 1350
1892 REM GOSUB 1450
1898 GOTO 1100 'PRINT REF DATA AND STOP

```

# LISTING OF FILT: RESPONSE OF FILTER TO PULSE OFFSET IN FREQUENCY

```

100 DEFINT I-N
110 PI=3.14159:DTR=PI/180:RTD=180/PI
120 DIM PLT$(130)
130 AST$="+":BLK$=" ":PDS$="."
140 FOR I=1 TO 130:PLT$(I)=BLK$:NEXT I
150 FOR I=3 TO 123 STEP 10:PLT$(I)=PDS$:NEXT I
200 DIM X(101),R(101),S(101),DB(101),F(10),A(10)
210 DIM TU(10),D(10),P(15),Q(15),TN(10)
220 DIM C(15),H(15),AM(10),AN(10),U(101),V(101)
250 PRINT:PRINT "BRANCH KEYS:"PRINT "    BRANCH A (AT 400):"
251 PRINT "        1    NEW FILTER"
252 PRINT "        2    NEW SIGNAL"
253 PRINT "        3    PRE-COMP AND PROCEED TO BRANCH B"
254 PRINT "        4    JUMP TO BRANCH B"
255 PRINT "        5    PRINT CASE DATA"
256 PRINT "        6    PRINT POLES AND ZEROES"
260 PRINT:PRINT "    BRANCH B (AT 2000):"
261 PRINT "        1    MOD DELTA-T AND/OR NR. OF POINTS"
262 PRINT "        2    GO COMPUTE (MORE) POINTS"
263 PRINT "        3    RESET T AND COMPUTE POINTS"
264 PRINT "        4    PRINT DATA"
275 PRINT "        5    JUMP TO BRANCH A"
276 PRINT "        6    PLOT LOG ON PRINTER"
299 PRINT
300 IBC=2:BWN=10:RPL=1:NPL=3:NBRK=4:FOFF=10:N3=20:KRIP=2
310 FOR I=1 TO NBRK:READ H7,H8:A(I)=H8:TU(I)=H7:NEXT I
320 DATA 0,0,.02,1,.2,1,.22,0
350 GOTO 810
400 INPUT "BRANCH A";L1:ON L1 GOTO 600,1000,1300,2000,1100,1200:PRINT
600 INPUT "BUTTERWORTH/CHEBYSHEV (1/2),BW(MHZ),NR. POLES";IBC,BWN,NPL
602 IF IBC=2 GOTO 800
610 FOR I=1 TO NPL
620 F2=(2*I-1)*PI/(2*NPL)
630 P(I)=-SIN(F2):Q(I)=COS(F2):NEXT I
640 R9=1:R0=0:BW=BWN:GOTO 400
800 INPUT "RIPPLE(DB),BW TO BE 3DB OR RIPPLE (1/2)";RPL,KRIP
810 BW=BWN:IF KRIP=2 GOTO 870
820 Q1=1/(10^(RPL/10)-1):Q2=SQR(Q1)
830 Q3=LOG(Q2+SQR(Q1-1))/NPL:Q4=.5*(EXP(Q3)+EXP(-Q3)):BW=BWN/Q4
870 E1=SQR(10^(RPL/10)-1):B3=1/E1:A3=SQR(1+1/(E1*E1))
880 A1=(A3+B3)^(1/NPL):D1=(A3-B3)^(1/NPL)
890 G2=.5*(A1-D1):D2=.5*(A1+D1):R9=1
900 FOR I=1 TO NPL
910 F2=(2*I-1)*PI/(2*NPL):P0=-SIN(F2)*G2:Q0=COS(F2)*D2
920 P(I)=P0:Q(I)=Q0:R9=R9/(P0*P0+Q0*Q0):NEXT I
930 R9=SQR(R9):R0=8.68589*LOG(R9)
940 N4=NPL MOD 2:IF N4=1 GOTO 400
950 R0=R0+RPL:R9=10^(RPL/20):GOTO 400
1000 PRINT:INPUT "NR. OF ENV. BREAKS,CARRIER OFFSET";NBRK,FOFF
1010 FOR I=2 TO NBRK
1020 PRINT "BRK ";I;:INPUT "TIME,AMPL";H7,H8
1030 A(I)=H8:TU(I)=H7:NEXT I
1040 TU(1)=0:A(1)=0

```

```

1050 IF A(NBRK)=0 THEN PRINT:GOTO 400
1060 INPUT "PULSE NOT TERMINATED;TRY AGAIN/CONT (1/2)";I2
1070 IF I2=1 GOTO 1010
1072 PRINT:GOTO 400
1080 FOR I=2 TO NBRK
1082 IF TU(I-1)>=TU(I) THEN PRINT "T'S MUST INCREASE":GOTO 1010
1090 NEXT I
1100 LPRINT:ON IBC GOTO 1110,1130
1110 LPRINT "BUTTERWORTH":LPRINT " NR. OF POLES";NPL
1120 LPRINT " 3 DB BW ";BW:GOTO 1160
1130 LPRINT "CHEBYSHEV":LPRINT " NR. OF POLES";NPL
1140 LPRINT " RIPPLE ";RPL;"DB"
1142 IF KRIP=1 THEN LPRINT " 3 DB BW ";BWN
1150 LPRINT " RIPPLE BW ";BW
1160 LPRINT:LPRINT "SIGNAL:"
1170 LPRINT " CARRIER OFFSET";FOFF
1172 LPRINT " ENVELOPE BREAKS:"
1174 LPRINT " TIME","AMPLITUDE"
1180 FOR I=1 TO NBRK
1182 LPRINT TU(I),A(I):NEXT I:LPRINT:GOTO 400
1200 LPRINT "POLES:"
1210 FOR I=1 TO NPL
1220 LPRINT P(I),Q(I):NEXT I:LPRINT:GOTO 400
1300 FOR I=2 TO NBRK
1310 TN(I)=TU(I)*PI*BW:NEXT I
1350 FOR I=2 TO NBRK
1360 F(I)=(A(I)-A(I-1))/(TN(I)-TN(I-1)):NEXT I
1370 D(1)=F(2)
1380 FOR I=2 TO NBRK-1
1390 D(I)=F(I+1)-F(I):NEXT I:D(NBRK)=-F(NBRK)
1400 W1=2*FOFF/BW:T1=2/(N3*BW):T2=.434294*LOG(T1)
1410 T3=INT(T2):T4=T2-T3
1420 IF T4<.15 GOTO 1460
1430 IF T4>.85 GOTO 1470
1440 IF T4>.5 GOTO 1480
1450 T5=T2*10^T3:GOTO 1490
1460 T5=10^T3:GOTO 1490
1470 T5=10^(T3+1):GOTO 1490
1480 T5=5*10^T3
1490 T6=T5*PI*BW
1500 VI5=1:VJ5=0:B5=0:D5=0
1510 FOR I=1 TO NPL
1520 E5=-P(I):F5=W1-Q(I)
1530 VI6=VI5:Q6=VJ5:VI7=E5:Q7=F5
1532 GOSUB 9100
1534 VI5=VI8:VJ5=Q8
1540 VI6=1:Q6=0:VI7=E5:Q7=F5:GOSUB 9130:B5=B5+VI8:D5=D5+Q8
1550 G5=1:H5=0
1600 FOR II=1 TO NPL
1602 IF II=I GOTO 1640
1610 VI7=P(I)-P(II):Q7=Q(I)-Q(II)
1620 VI6=G5:Q6=H5:GOSUB 9100:G5=VI8:H5=Q8
1640 NEXT II
1650 VI6=-E5:Q6=-F5:VI7=-E5:Q7=-F5:GOSUB 9100:VI6=VI8:Q6=Q8
1660 VI7=G5:Q7=H5:GOSUB 9100:VI6=1:Q6=0:VI7=VI8:Q7=Q8

```

```

1670 GOSUB 9130:C(I)=VI8:H(I)=Q8:NEXT I
1700 VI6=1:Q6=0:VI7=VI5:Q7=VJ5:GOSUB 9130:A5=VI8:C5=Q8
1710 A0=SQR(A5*A5+C5*C5)/R9
1720 LPRINT "MAG/R9";A0;" OR ";8.6859*LOG( 7);"DB"
1740 S5=0:S6=0
1750 FOR I=1 TO NBRK
1760 S5=S5+D(I):S6=S6+D(I)*TN(I):AM(I)=S5:AN(I)=S6:NEXT I
1762 N6=40:K2=1:T7=0:KFRST=1
1770 LPRINT "NR. OF INTERVALS";N6;" DELTA-T";T5:N6=N6+1
1780 PRINT "DELTA T ";T5;" NR. STEPS ";N6-1
2000 INPUT "BRANCH B";L1
2010 ON L1 GOTO 2100,2200,2150,6000,400,3000
2100 INPUT "DELTA-T,NR OF INTERVALS";T5,N6:T6=T5*PI*BW:GOTO 1770
2150 K2=1:T7=0:KFRST=1
2200 AMAX=0:J1=K2:R(1)=0:S(1)=0:DB(1)=-90
2210 FOR JJ=1 TO N6
2220 T8=T6*(JJ-1)+T7:X(K2)=K2-1
2230 IF K2=1 GOTO 2520
2240 K4=1
2242 K0=K4:IF TN(K4)>=T8 GOTO 2300
2250 K4=K4+1:IF K4<=NBRK GOTO 2242
2260 L=K0:GOTO 2310
2300 L=K0-1
2310 VI6=T8-B5:Q6=-D5:VI7=AM(L):Q7=0:GOSUB 9100
2320 VK5=VI8-AN(L):VL5=Q8:AM5=0:AN5=0
2340 FOR I=1 TO NPL:Q5=0:R5=0
2360 FOR K6=1 TO L
2370 H7=TN(K6):H8=T8-H7:H9=D(K6)
2380 VI6=P(I)*H8:Q6=Q(I)*H8+W1*H7:GOSUB 9510
2390 Q5=Q5+H9*VI8:R5=R5+H9*Q8:NEXT K6
2400 VI6=Q5:Q6=R5:VI7=C(I):Q7=H(I):GOSUB 9100
2410 AM5=AM5+VI8:AN5=AN5+Q8:NEXT I
2420 W=W1*T8:VI6=0:Q6=W:GOSUB 9510:Y5=VI8:Z5=Q8
2430 VI6=Y5:Q6=Z5:VI7=A5:Q7=C5:GOSUB 9100:VI7=VI8:Q7=Q8
2440 VI6=VK5:Q6=VL5:GOSUB 9100:VI6=VI8+AM5:Q6=Q8+AN5
2450 H7=SQR(VI6*VI6+Q6*Q6)/R9
2460 R(K2)=VI6/R9:S(K2)=Q6/R9
2500 DB(K2)=8.68589*LOG(H7)
2510 IF AMAX<H7 THEN AMAX=H7
2520 K2=K2+1:NEXT JJ
2530 DBX=8.68589*LOG(AMAX)
2540 PRINT:PRINT "MAX ";AMAX;" MAX (DB)";DBX
2550 K7=0:T7=T8:K2=K2-1:GOTO 2000
3000 LPRINT CHR$(15):IF KFRST=0 GOTO 3020
3002 LPRINT " -50 -40 -30";
3004 LPRINT " -20 -10
0 DB"
3020 FOR I=J1-KFRST+1 TO J1+N6-1
3030 WX=DB(I):IF WX<-56 THEN WX=-56
3040 IF WX>9 THEN WX=9
3050 NPLT=INT(WX*2+.5)+113:IF NPLT<1 THEN NPLT=1
3060 IF NPLT>130 THEN NPLT=130
3070 TEMP$=PLT$(NPLT)
3080 PLT$(NPLT)=AST$
3090 FOR J=1 TO 130:LPRINT PLT$(J);:NEXT J:LPRINT

```

```

3100 PLT$(NPLT)=TEMPS
3110 NEXT I:LPRINT CHR$(146):KFRST=0:GOTO 2000
6000 LPRINT
6010 FOR I=J1 TO J1+N6-1:LPRINT USING "###      ";I;
6020 LPRINT USING "###.###      ";R(I),S(I),DB(I):NEXT I:GOTO 2000
9100 GOSUB 9600
9102 VI8=VI6*VI7-Q6*Q7:Q8=Q6*VI7+VI6*Q7:RETURN
9130 GOSUB 9600
9140 W8=VI7*VI7+Q7*Q7:IF W8>1E-18 GOTO 9180
9150 VI8=1E+18:Q8=1E+18:Z8=1:RETURN
9180 Z8=0:VI8=(VI6*VI7+Q6*Q7)/W8:Q8=(Q6*VI7-VI6*Q7)/W8:RETURN
9510 IF ABS(VI6)>.000001 GOTO 9530
9520 W8=1+VI6:GOTO 9560
9530 IF ABS(VI6)<69 GOTO 9550
9540 IF SGN(VI6)>0 GOTO 9546
9542 W8=1E-18:GOTO 9560
9546 W8=1E+18:GOTO 9560
9550 W8=EXP(VI6)
9560 VI8=W8*COS(Q6):Q8=W8*SIN(Q6):RETURN
9600 IF ABS(VI6)<1E-18 THEN VI6=0
9620 IF ABS(Q6)<1E-18 THEN Q6=0
9640 IF ABS(VI7)<1E-18 THEN VI7=0
9660 IF ABS(Q7)<1E-18 THEN Q7=0
9670 RETURN

```

APPENDIX F

ANALYSIS OF FILTER RESPONSE  
TO OFF-SET PULSE SIGNALS

## 1. INTRODUCTION

The objective is an efficient method of computing the transient output envelope of a band-pass filter when driven by a pulsed signal with a carrier frequency off-set from the filter center frequency, either in-band or out-of-band. The filter should be realizable and easily specified, and the pulsed signal should also be readily defined and reasonably general. In the program as developed, the filter can be defined by its bandwidth, number of poles, and ripple. For the Butterworth (zero ripple) case, the bandwidth is at 3 dB down; for Tchebycheff filters, the bandwidth is at the ripple points (the same level as the valleys). The pulse is defined by its carrier off-set and a set of times and amplitudes specifying a piecewise linear envelope starting at zero amplitude at zero time. Real pulses may have significant intra-pulse FM, but this is beyond the scope of the present effort.

The approach is to find the poles of the La Place transform of the output (product of transfer function and transform of the input) and then (by the residue theorem) evaluate the inverse transform as the sum of the residues, in these poles, of the integrand appearing in the defining integral; this integrand is the above product times  $\exp(st)$ . This computation is actually done for the low-pass prototype; the RF carrier offset is used as the low-pass carrier frequency. This is strictly valid only for small percentage bandwidth.

The program can be expanded to cover Bessel function filters and elliptic function (Cauer) filters.

## 2. TRANSFER FUNCTION

For Butterworth and Tchebycheff filters, the transfer function contains only poles and is therefore of the form

$$H(s) = \frac{1}{\prod_{i=1}^n (s - p_i)}$$

for poles  $p_1$  to  $p_n$ . The poles of a Butterworth filter with 3 dB point at  $\omega = 1$  are located (for pole  $i$ ) at

$$\text{Re}(p_i) = -\sin\left(\frac{(2i-1)\pi}{2n}\right)$$

$$\text{Im}(p_i) = \cos\left(\frac{(2i-1)\pi}{2n}\right)$$

(See D. R. J. White, "Handbook on Electrical Filters", Section 4.1)

For a Tchebycheff filter, the poles can be found as follows. (Same reference, Section 4.2). The power response is  $1/[1 + \epsilon^2 T_n^2(\omega)]$  where  $T_n(\omega)$  is the Tchebycheff polynomial having a value of 1 at the valley points. This defines  $1/(1 + \epsilon^2)$  as the relative response at the valleys, or  $10^{-r/10}$  where  $r$  is the ripple (dB). Therefore

$$\epsilon^2 = 10^{r/10} - 1$$

Define:

$$\beta \triangleq 1/\epsilon$$

$$\alpha \triangleq 1 + 1/\epsilon^2$$

$$\vartheta \triangleq [(\alpha + \beta)^{1/n} - (\alpha - \beta)^{1/n}]/2$$

$$\delta \triangleq [(\alpha + \beta)^{1/n} + (\alpha - \beta)^{1/n}]/2$$

Then:

$$\operatorname{Re}(p_1) = -\delta \sin \left( \frac{(2i-1)\pi}{2n} \right)$$

$$\operatorname{Im}(p_1) = \delta \cos \left( \frac{(2i-1)\pi}{2n} \right)$$

### 3. LA PLACE TRANSFORM OF INPUT PULSE

The input envelope is shown in Figure F-1, defined by breaks at times  $T_1$  to  $T_m$  at amplitudes  $A_1$  to  $A_m$ ;  $T_1$  and  $A_1$  are always to be zero and  $A_m$  is also normally zero. This is a superposition of ramps starting at the break times  $T_j$  and of varying slopes, each of which is the change in net slope at the break. These ramp slopes  $d_j$  are easily found from the A's and the T's. The transform of a ramp of slope  $d_j$  starting at  $T_j$  is

$$\frac{d_j \exp(-ST_j)}{S^2}$$

giving the envelope transform as

$$\sum_{j=1}^m \frac{d_j \exp(-ST_j)}{S^2}$$

Multiplying the envelope by a carrier  $\exp j\Delta\omega t$  where  $\Delta\omega$  is the off-set (or the signal carrier itself for the low-pass case) requires  $S$  to be replaced by  $S - j\Delta\omega$ , giving the signal transform as

$$S(s) = \sum_{j=1}^m \frac{d_j \exp[-T_j(S - j\Delta\omega)]}{(S - j\Delta\omega)^2}$$

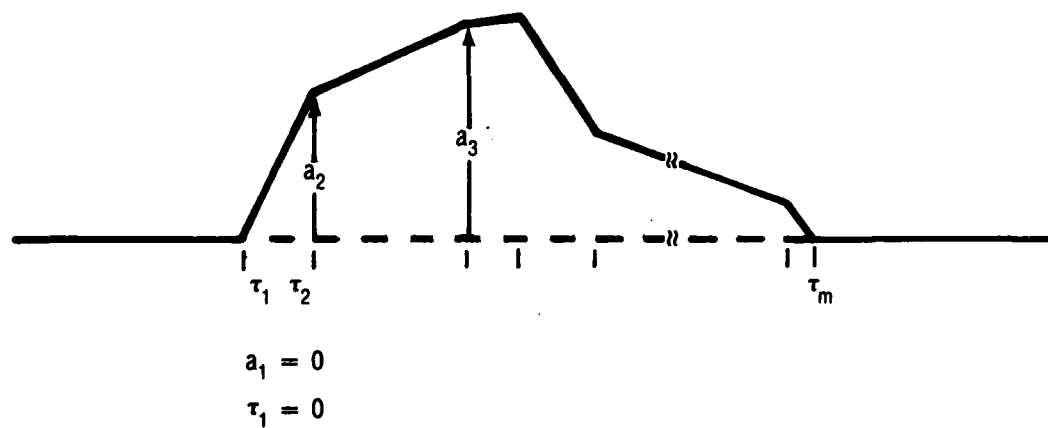


FIG. F-1.

FIGURE F-1 SIGNAL ENVELOPE

#### 4. INVERSE LA PLACE TRANSFORM

The response  $r(t)$  is the inverse transform of  $H(s) S(s)$

$$r(t) = \frac{1}{2\pi j} \int_{C-J^{\infty}}^{C+J^{\infty}} H(s) S(s) \exp(st) ds$$

The contour can be closed by a semi-circle in the left half plane with radius approaching  $\infty$  without affecting the integral since all singularities of the integrand are finite, isolated poles. The integral is then  $2\pi j$  times the sum of the residues of the integrand in the poles. Using the expressions above, the integrand is

$$\frac{\exp(st)}{(s - j\Delta\omega)^2 \prod_{i=1}^m (s - p_i)} \sum_{k=1}^m d_k \exp[-T_k(s - j\Delta\omega)]$$

This has poles  $p_i$  and a double pole at  $j\Delta\omega$ .

The residue in the double pole is found by deleting the factor  $(s - j\Delta\omega)$  from the denominator, differentiating the resulting expression (analytic in the neighborhood of  $j\Delta\omega$ ) and evaluating the derivative at  $s = j\Delta\omega$ . The result is the coefficient of the first degree term in  $(s - j\Delta\omega)$  in the Taylor expansion about  $j\Delta\omega$  of the analytic expression. (Recall the ordinary Taylor expansion  $f(x) = f(x_0) + f'(x_0)(x - x_0) + f''(x_0)(x - x_0)^2/2! + \dots$ .) Dividing this Taylor expansion by  $(s - j\Delta\omega)$  gives the Laurent series about  $j\Delta\omega$ , whose term of degree  $-1$  in  $(s - j\Delta\omega)$  has the same coefficient. This coefficient is therefore the residue in the double pole. The analytic expression is

$$\frac{\exp(st)}{\prod_{i=1}^m (s - p_i)} \sum_{k=1}^m d_k \exp[-T_k(s - j\Delta\omega)]$$

The derivative can be found by the useful formula:

$$d\left(\frac{u v w \dots}{r s t \dots}\right) = \frac{u v w \dots}{r s t \dots} \left(\frac{du}{u} + \frac{dv}{v} + \frac{dw}{w} \dots - \frac{dr}{r} - \frac{ds}{s} - \frac{dt}{t} \dots\right)$$

This gives

$$\frac{\exp(st)}{\prod_{i=1}^n (s-p_i)} = \sum_{k=1}^m d_k \exp[-T_k(s - j\Delta\omega)] \left(t - T_k - \sum_{i=1}^n \frac{1}{s - p_i}\right)$$

At  $s = j\Delta\omega$  this becomes

$$\frac{\exp j\Delta t}{\prod_{i=1}^n (j\Delta\omega - p_i)} = \sum_{k=1}^m d_k \left(t - T_k - \sum_{i=1}^n \frac{1}{j\Delta\omega - p_i}\right)$$

This can be partly pre-computed in advance of computing a time series. Define:

$$A \triangleq \frac{1}{\prod_{i=1}^n (j\Delta\omega - p_i)}$$

$$B \triangleq \sum_{i=1}^n \frac{1}{j\Delta\omega - p_i}$$

Then this part of the output (due to the double pole) becomes:

$$\exp j\Delta\omega t \left[ A \sum_{k=1}^m d_k (t - T_k - B) \right]$$

A given term  $k$  "turns on" only when  $t$  passes  $T_k$  so that the summation in  $k$  is better written as

$$\sum_{h=1}^l d_k (t - T_k - B) \quad \begin{array}{l} T_l < t \\ T_{l+1} \geq t \end{array}$$

The summation is carried only so far, depending on  $t$ . The summation can be written:

$$\sum d_k T_k + (t - B) \sum d_k$$

These summations for  $l = 1$  to  $m$  can be precomputed and stored. Define:

$$F_l \triangleq \sum_{k=1}^l d_k T_k$$

$$G_l \triangleq \sum_{k=1}^l d_k$$

Then the part of the output due to the double pole at  $j\Delta\omega$  is

$$\exp j\Delta\omega t A [F_l + (t - B) G_l] \quad \begin{array}{l} T_l < t \\ T_{l+1} \geq t \end{array}$$

Depending on  $t$ , the proper sums are picked up from the stored arrays as the time series progresses.

For each filter pole,  $p_i$ , the procedure is similar except that differentiation is omitted; the coefficient of the inverse first power in the Laurent series (the residue) is merely the first term of the Taylor expansion. For each pole  $p_i$ , delete the factor  $(s - p_i)$ :

$$\frac{\exp st}{(s - j\Delta\omega)^2 \prod_{\substack{g=1 \\ g \neq i}}^n (s - p_g)} \sum_{k=1}^m d_k \exp[-T_k(s - j\Delta\omega)]$$

Evaluate at  $s = p_i$ :

$$\frac{\exp p_i t}{(p_i - j\Delta\omega) \prod_{\substack{g=1 \\ g \neq i}}^n (p_i - p_g)} \cdot \sum_{k=1}^l d_k \exp[-T_k(p_i - j\Delta\omega)] \quad \begin{matrix} T_l < t \\ T_{l+1} = t \end{matrix}$$

As above, each term in  $k$  turns on when  $t$  passes  $T_k$  so that the summation in  $k$  is limited by the value of  $t$ . The above expression, in principle, permits pre-computation and storage of all except the factor  $\exp p_i t$  in array  $D_{il}$  which is not a function of time. Unfortunately, the two exponentials can go to extremely small and extremely large values respectively when the real parts of the arguments get too large in magnitude (about 175 in the 370 computer). Consequently, the pre-computation of  $D_{il}$  tended to "blow" the exponential for large  $T_k$ , and  $\exp p_i t$  also gave trouble in the time series computation. Therefore, the computation was changed in accord with the rewritten expression:

$$\sum_{i=1}^n \frac{1}{(p_i - j\Delta\omega)^2 \prod_{\substack{g=1 \\ g \neq i}}^n (p_i - p_g)} \sum_{k=1}^l d_k \exp[p_i(t - T_k) + j\Delta\omega T_k] \quad \begin{matrix} T_l < t \\ T_{l+1} = t \end{matrix}$$

The factor between summation signs can still be precomputed and stored as  $C_i$  but the summation in  $k$  is computed in full at each  $t$ . Now however, the partial cancellation of a very small and a very large exponential when multiplied together is replaced and accomplished by the partial cancellation of their summed arguments before evaluating the single exponential.

Finally, combining the contributions from all the poles,

$$r(t) = \exp j\Delta\omega t \quad A[F + (t - B) G] \quad \begin{matrix} T_\ell < t \\ T_{\ell+1} \geq t \end{matrix}$$

$$+ \sum_{i=1}^n C_i \sum_{k=1}^1 d_k \exp[\pi i(t - T_k) + j\Delta\omega T_k]$$

The real part of this complex output is the physical output of the filter. However, this would exhibit the coarse intra-envelope carrier at the offset frequency, which is not representative of the envelope at the bandpass frequency and would require small  $\Delta t$  steps for reasonable interpretability. Therefore,  $|r(t)|$  is a better output variable.

## 5. COMPUTER PROGRAM

The program has been set up to provide easy inputting of parameters and flexibility of outputting. Output is plotted either in linear envelope amplitude or in dB and is normalized to the steady state peak response of the filter. Maximum output is printed before plotting and plot scales can be modified as

as appropriate. However, default parameters are provided, including a 3-pole Tchebycheff filter with 1 dB ripple and a nearly rectangular signal offset by the ripple bandwidth of the filter, plotted at one sample per cycle of the half-bandwidth.

APPENDIX G

EXAMPLE SIMULATION PLOTS

## INTRODUCTION

The simulation plotted video for the composite reflected signal from a number of point scatters and for the composite signal after cancellation parameters were determined. Example plot pairs are included below.

## REPRESENTATIVE SIMULATION PLOTS

The simulations used for this study presented results as video versus time plots before and after cancellation. Examples are presented as Figures G-1 to G-4. The top plot in each figure presents logarithmic video amplitude (dB) versus time without benefit of cancellation. This signal is a composite of reflections from a number of point scatterers. Amplitude is normalized for a peak value of 0 dB. The lower plot shows video after cancellation using the same amplitude and time scales.

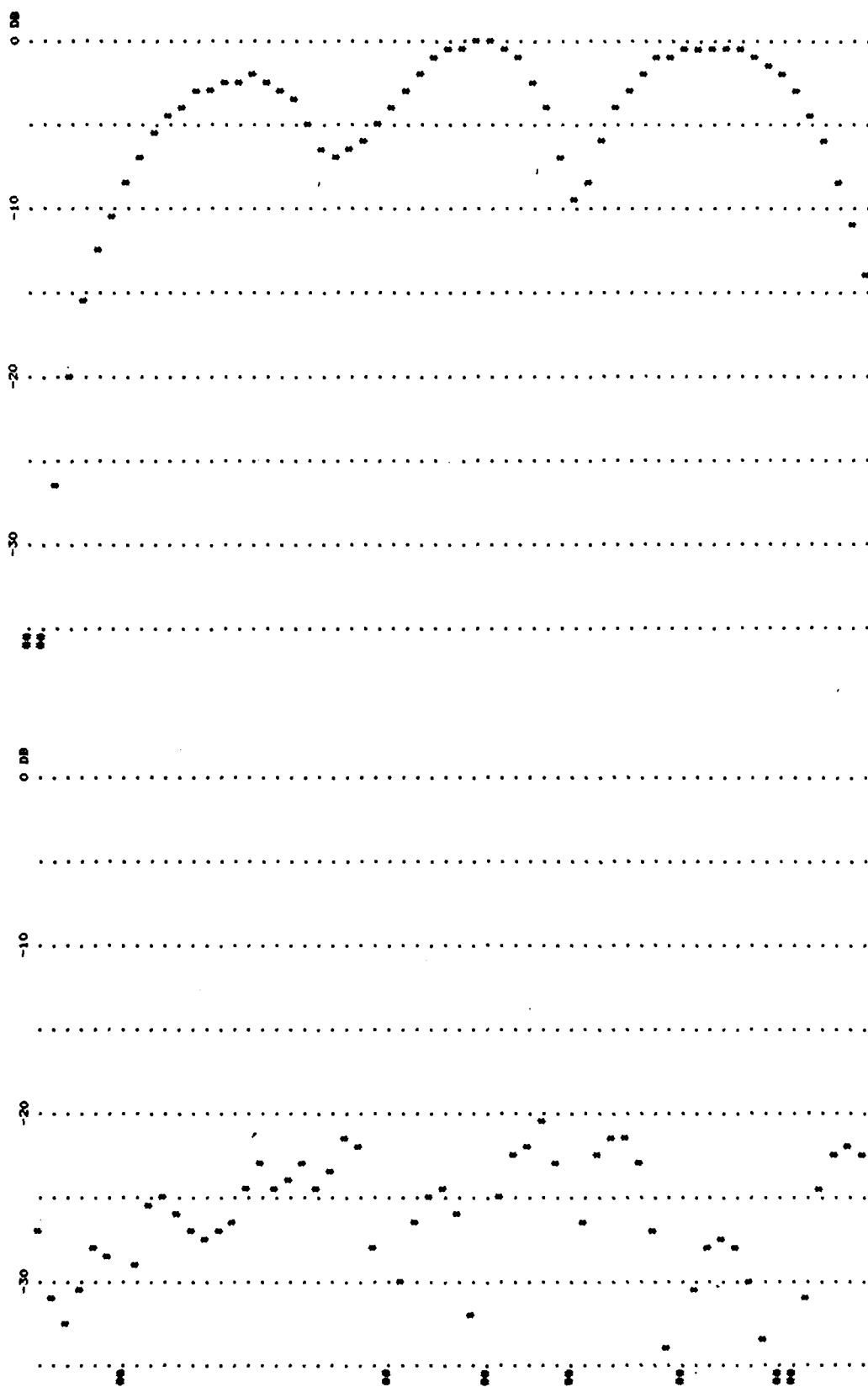


FIGURE G-1 EXAMPLE SIMULATION OUTPUT 1-LOG VIDEO VERSUS TIME  
BEFORE AND AFTER CANCELLATION

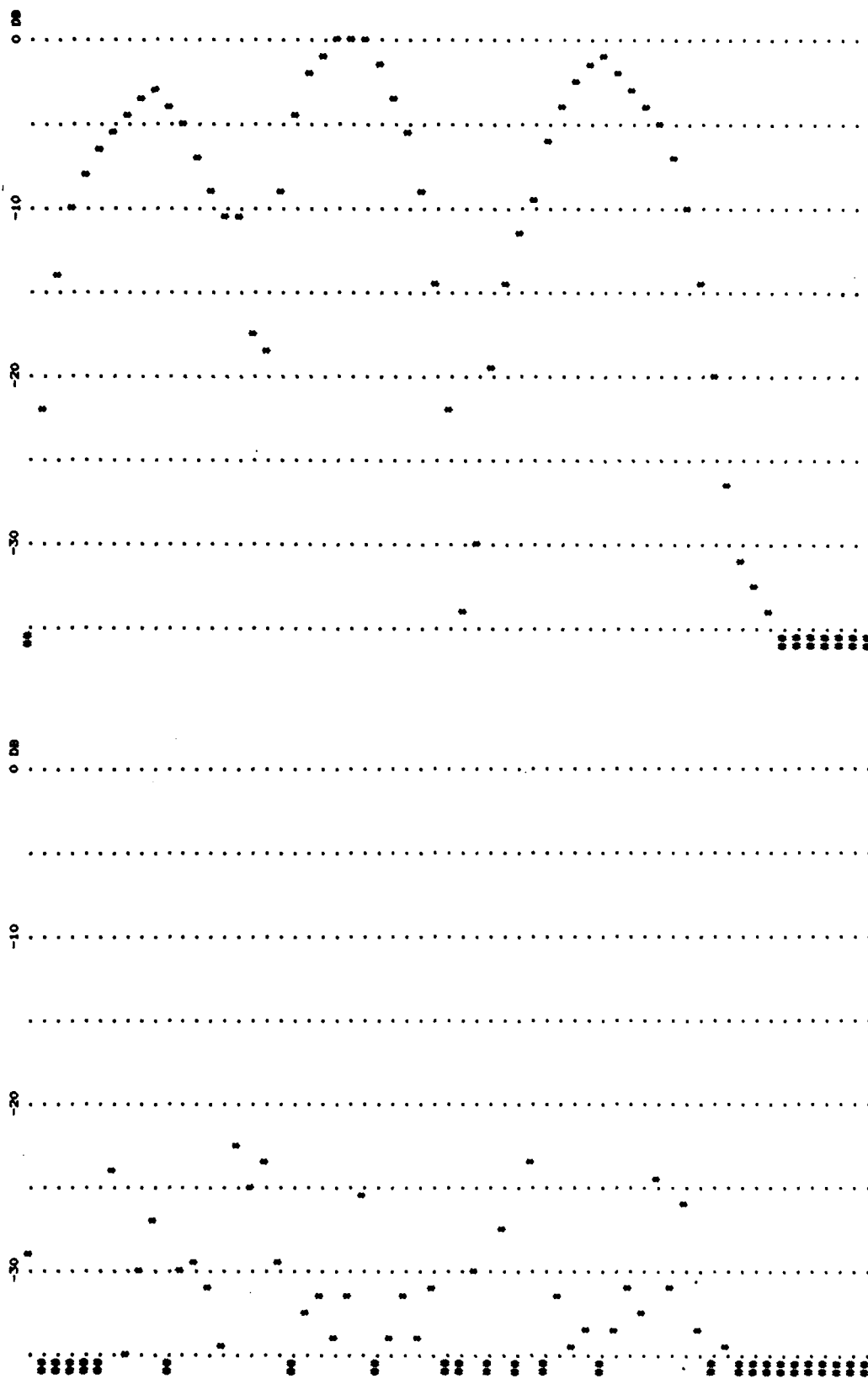


FIGURE G-2 EXAMPLE SIMULATION OUTPUT 2-LOG VIDEO VERSUS TIME  
BEFORE AND AFTER CANCELLATION

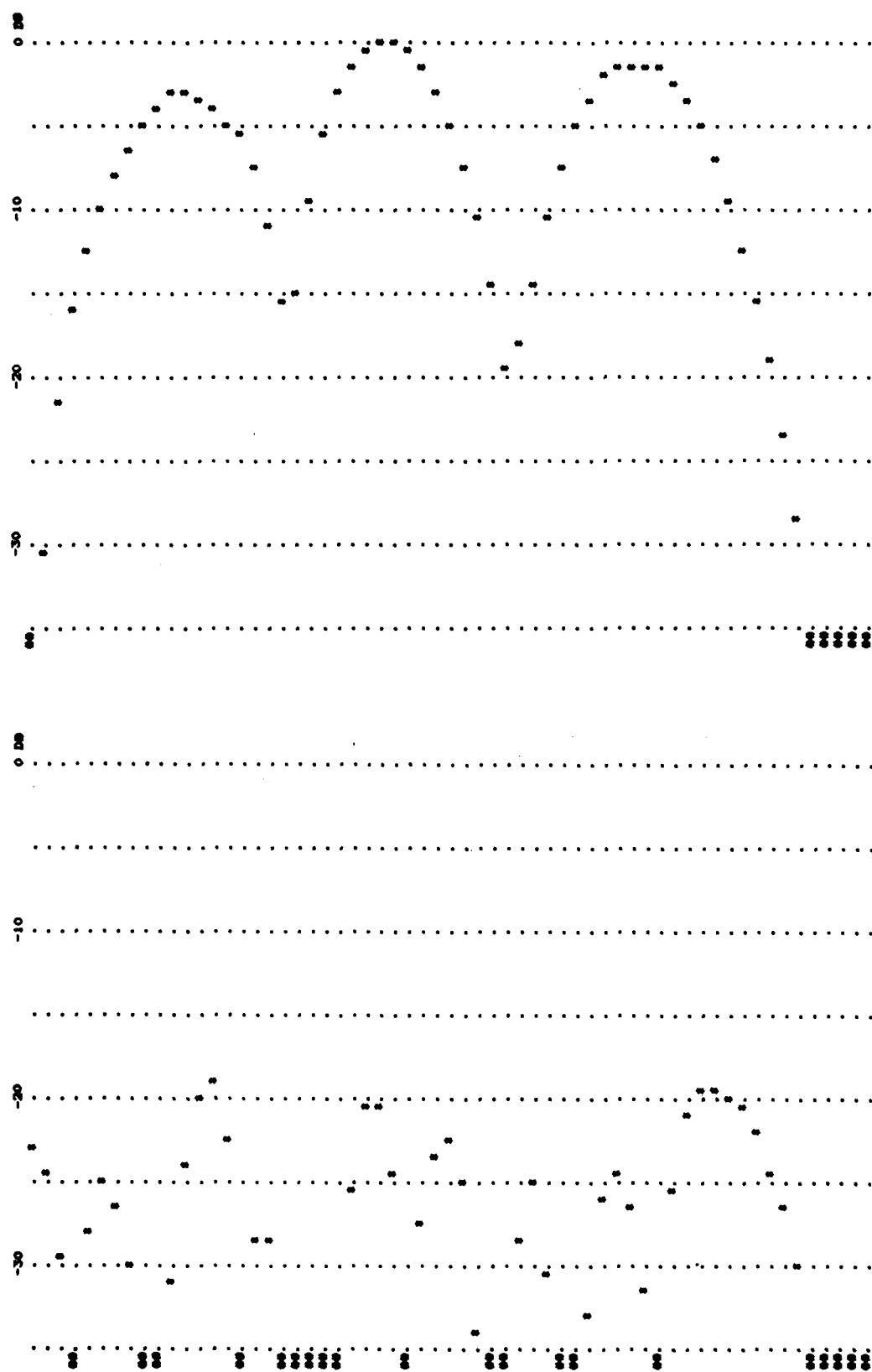


FIGURE G-3 EXAMPLE SIMULATION OUTPUT 3-LOG VIDEO VERSUS TIME  
BEFORE AND AFTER CANCELLATION

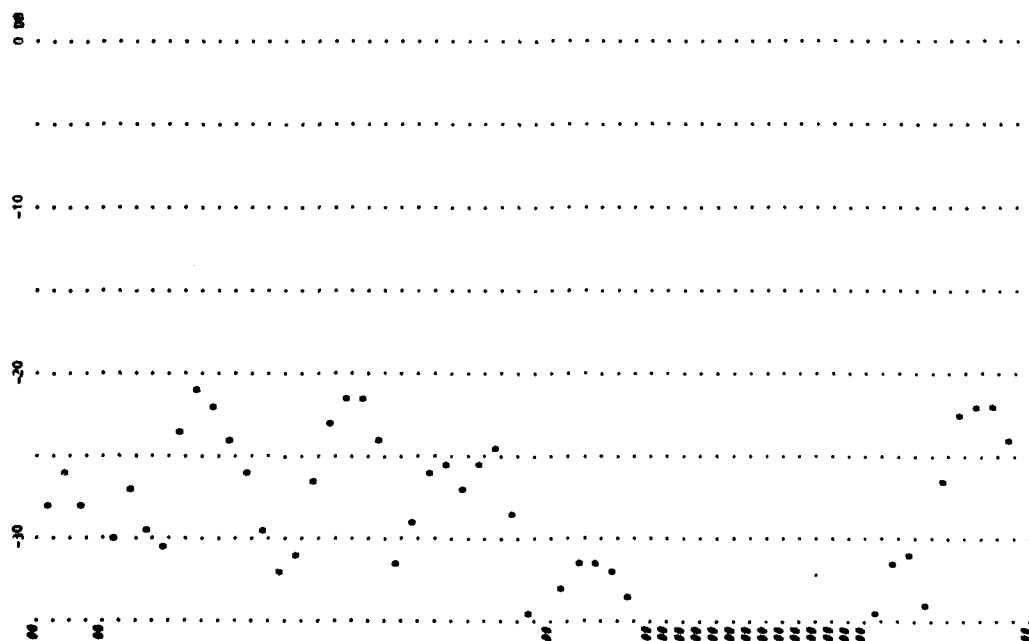
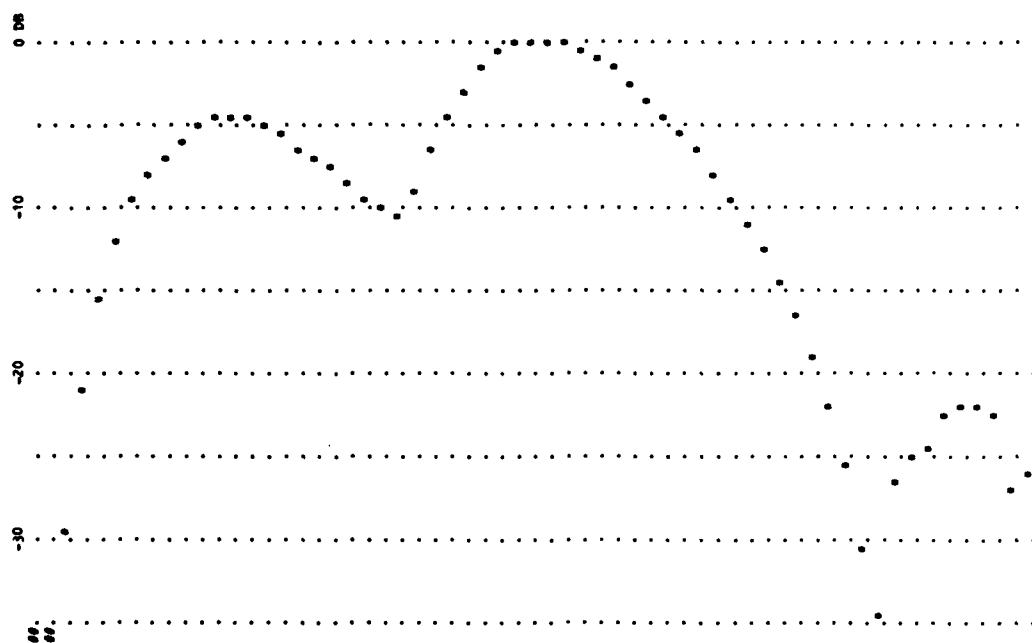


FIGURE G-4 EXAMPLE SIMULATION OUTPUT 4-LOG VIDEO VERSUS TIME  
BEFORE AND AFTER CANCELLATION

**END**

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